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The Liège Orbital Solution Package

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ABSTRACT
We present the Liège Orbital Solution Package (LOSP), a numerical package that aims at computing the orbital parameters of spectroscopic binaries. The package deals with SB1 and SB2 systems and is able to adjust either circular or eccentric orbits through a weighted fit. The SB1 eccentric fit relies on the Wolfe et al. method. The SB2 eccentric orbits are adjusted using the Liège SB2 Orbital Solution Algorithm (LOSA), that uses an orthogonal regression technique and a modified version of the Wolfe at al. algorithm to derive self-consistent solutions for both components of the system. The SB1 circular orbits are fitted through a linear least square technique. Finally, the same approach as the one adopted in LOSA as been adapted to SB2 circular systems. It allows to generate self-consistent solutions for both components. As an additional capability, LOSP can perform an exploration of the parameter space along the period axis and, eventually, along the ratio of the secondary to primary radial velocities uncertainties. Beyond the standard error computations, LOSP further provides the opportunity to perform Monte-Carlo simulations on the basis of the best-fit solution. This option allows an independent and robust check of the accuracy of the determined parameters in the four considered cases (SB1 vs. SB2, circular vs. eccentric orbit).

Key words: Binaries: spectroscopic – Stars: fundamental parameters – Methods: numerical

1 INTRODUCTION

It has long been known that most stars in the Galaxy belong to binary or multiple systems. Among the various kinds of systems, spectroscopic binaries are considered as privileged laboratories to constrain the stars fundamental parameters. Through their gravitational interaction, they potentially offers to accurately measure the masses. In practice, the situation is unfortunately less simple as accurate masses and radii can only be obtained for SB2 eclipsing binaries. SB2 non eclipsing systems only yield the minimal masses $M\ \sin^3 i$ of each components (where i is the orbital inclination). Still the mass ratio of these systems can be measured independently of the inclination. Finally SB1 systems only provided a limited information on the stars weight through the so-called mass function $f_{\mathrm{mass}} = \frac{PK^3}{2\pi G}\left(1-e^2\right)^{3/2}$, where the notations used are defined in Table 1. However, even SB1 systems bring important constraints on the formation and evolution of binaries through e.g. the period vs. eccentricity diagram. Beside the intrinsic quality of the data, an important aspect of orbital solution fitting lies in an accurate estimation of the uncertainties that spoil the derived orbital parameters.

In this work, we present a package that allows to compute orbital solutions of spectroscopic binaries. Developed at the Liège University (Belgium), the package indifferently addresses SB1 or SB2 systems and deals with both circular or eccentric orbits. It only requires a first guess of the location of the orbital period in the parameter space, which can easily be obtained through an appropriate Fourier analysis (Heck et al. 1985, see also Gosset et al. 2001 for comments). The preliminary period value may then be improved either by a differential correction method (for eccentric systems) or through a global minimization of the χ^2 along this particular degree of freedom (both for eccentric or non eccentric systems). Finally, a particular attention has been given to the estimates of the errors. Beside a first estimate obtained through the theory of error propagation, the package also allows to perform Monte-Carlo simulations to estimate the confidence limits on the best-fit parameters.

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Table 1. Adopted notations for the orbital parameters of a massive binary system.

Notation	Description
P	Orbital period
i	System inclination
e	Orbit eccentricity [0,1]
T_0	Time of periastron passage ($e \neq 0$) or time of primary conjunction ($e = 0$)
ω	Longitude of the periastron ($e \neq 0$)
$K_{1,2}$	Radial velocity curve semi-amplitude for component 1, 2
$\gamma_{1,2}$	Apparent systemic radial velocity associated with component 1, 2
$a_{1,2}$	Semi-major axis of the orbit for component 1, 2. We also note $a = a_1 + a_2$
$M_{1,2}$	Mass of component 1, 2
$q = M_1/M_2$	Binary component mass ratio
$R_{\mathrm{RL}_{1,2}}$	Roche lobe radius associated with component 1, 2
$f_{\mathrm{mass}_{1,2}}$	Mass function associated with component 1, 2

2 AN OVERVIEW OF THE PACKAGE

As mentioned above, LOSP mainly handles four cases: SB1 and SB2 eccentric orbits as well as SB1 and SB2 circular orbits. We now briefly present the underlining methods adopted in each of the four cases.

2.1 SB1 eccentric orbit

The adopted algorithm is that of Wolfe et al. (1967), that relies on the Wilsing-Russell method (Wilsing 1894; Russell 1902) followed by a differential correction (D.C.). It requires a first guess of the orbital period that can, on request, be improved by the D.C. As an additional capability, LOSP proposes to explore the period axis in a given interval and with a given step and return the period value that minimized the χ^2 in the considered interval. The WHS67 method offers the advantage that no a priori estimate of the location of the solution in the parameter space is required nor is an exploration of the 5-dimension parameter space. Wolfe et al. (1967) stated that the method is robust up to eccentricities about 0.8 though no detailed testing was presented.

2.2 SB2 eccentric orbit

A dedicated algorithm for SB2 eccentric orbit has been developed and is known as the Liège SB2 orbital Solution Algorithm. Roughly speaking, it uses an orthogonal regression technique to fit the v_2 vs. v_1 relationship (which directly provides the mass ratio) qnd to transform the SB2 RV data set into a *fake* but equivalent SB1 RV data set. The latter is characterized by the same period, eccentricity, longitude of periastron and time of periastron passage as the real SB2 system. The methods then applies a modified version of the WHS67 method to provide the orbital elements of the equivalent SB1 system from which the the final SB2 solution can be derived. The Liège SB2 algorithm offers the advantage to provide self-consistent SB2 solutions while preserving much of the properties of the WHS67 method. In particular, only a first guess of the period is needed and no fastidious exploration of the 8-dimension parameters space is required. Details of the algorithm are presented in Sana (2006).

2.3 SB1 circular orbit

For SB1 circular orbits, LOSP performs a linear least square fit of the system RV equation:

$$v(t_i) = \gamma + K \sin \phi_i, \tag{1}$$

where $\phi_i = 2\pi \frac{t_i - T_0}{P}$ is the phase angle measured from the time of primary conjunction. This problem has three unknowns: γ, K and T_0. Let us define $\phi' = 2\pi \frac{t_i - t_{\mathrm{arb}}}{P}$ where t_{arb} is an arbitrary time. The phase angle can now be rewritten: $\phi_i = \phi'_i + \omega = 2\pi \frac{t_i - t_{\mathrm{arb}}}{P} + 2\pi \frac{t_{\mathrm{arb}} - T_0}{P}$. Hence Eq. 1 can be rewritten:

$$v(t_i) = \gamma + K \cos \omega \sin \phi'_i + K \sin \omega \cos \phi'_i \tag{2}$$

that can now be solved using a linear least-square techniques with unknowns: γ, $K \cos \omega$ and $K \sin \omega$. The values of K and T_0 follows from the latter determination. As in the eccentric case, LOSP offers in addition the option to scan a given period interval to search for the best period values.

2.4 SB2 circular orbit

SB2 circular orbits are handle pretty much the same way as eccentric orbits, though the equation system is this time:

$$\left\{ \begin{array}{l} v_1 = \gamma_1 + K_1 \sin\phi \\ v_2 = \gamma_2 + K_2 \sin(\phi + \pi) \end{array} \right. \tag{3}$$

which, for a given value of the period, has five unknowns: γ_1, γ_2, K_1, K_2 and T_0. It is easy to show that

$$\frac{v_1(\phi) - \gamma_1}{K_1} = -\frac{v_2(\phi) - \gamma_2}{K_2} \tag{4}$$

or equivalently

$$v_2(\phi) = b + c v_1(\phi) \tag{5}$$

with $b = \gamma_2 - \frac{K_2}{K_1}\gamma_1$ and $c = -\frac{K_2}{K_1}$. This latter equation is linear in the parameters b and c and, given a set of $k = 1...N$ observation couples $(v_1(\theta_k), v_2(\theta_k))$, the system can be solved using the same orthogonal (because both v_1 and v_2 are spoiled by errors) regression technique as in the SB2 eccentric case. On can apply the following transformation in the radial velocity space :

$$\left\{ \begin{array}{l} v_1^* = \sqrt{-c}\left(v_1 - \frac{b}{1-c}\right) \\ v_2^* = \frac{1}{\sqrt{-c}}\left(v_2 - \frac{b}{1-c}\right) \end{array} \right. \tag{6}$$

to create the following equation system:

$$\left\{ \begin{array}{l} v_1^* = \Gamma + \mathcal{K}\cos\omega + \mathcal{K}\sin\omega \\ v_2^* = -\Gamma - \mathcal{K}\cos\omega - \mathcal{K}\sin\omega \end{array} \right. \tag{7}$$

with $\Gamma = \frac{\sqrt{-c}}{1-c}(\gamma_1 - \gamma_2)$ and $\mathcal{K} = \sqrt{K_1 K_2}$. The latter system is now linear in the parameters Γ, $\mathcal{K}\cos\omega$ and $\mathcal{K}\sin\omega$ and can be solved using a classical linear least square. The orbital solution of the SB2 system is then easily found from the best-fit values Γ, $\mathcal{K}\cos\omega$ and $\mathcal{K}\sin\omega$ and from the results of the orthogonal regression that yields b and c.

3 WHERE TO GET THE PACKAGE ?

The LOSP package consists of a FORTRAN-77 coded set of programs/routines handled transparently (from the user point of view) through a single BASH script. As a last capability, phase diagrams of the data and of the best-fit RV-curves are displayed using SUPER MONGO. This allows a visual inspection of the adjustment results. LOSP is quick, light ($\sim$ 200 Ko), easy to install and to use. It can be downloaded as a single archive file at ftp://arachnos.astro.ulg.ac.be/pub/users/sana/LOSP/ ($\sim$ 30 Ko). More detailed documentation is also included in the archive.

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APPENDIX A: TRANSFORMATION IN THE RV SPACE FOR SB2 CIRCULAR ORBIT

Starting from the RV equations for the two components of a circular SB2 binary :

$$\left\{ \begin{array}{lcl} v_1 & = & \gamma_1 + K_1 \cos(\theta + \phi) \\ v_2 & = & \gamma_2 - K_2 \cos(\theta + \phi) \end{array} \right. \quad \text{(A1)}$$

$$\left\{ \begin{array}{lcl} \left(v_1 - \frac{b}{1-c}\right)\sqrt{-c} & = & \left(\gamma_1 - \frac{\gamma_2 - c\gamma_1}{1-c}\right)\sqrt{-c} + K_1\sqrt{-c}\cos(\theta+\phi) \\ \left(v_2 - \frac{b}{1-c}\right)\frac{1}{\sqrt{-c}} & = & \left(\gamma_2 - \frac{\gamma_2 - c\gamma_1}{1-c}\right)\frac{1}{\sqrt{-c}} - K_2\frac{1}{\sqrt{-c}}\cos(\theta+\phi) \end{array} \right. \quad \text{(A2)}$$

$$\left\{ \begin{array}{lcl} \left(v_1 - \frac{b}{1-c}\right)\sqrt{-c} & = & \frac{\gamma_1 - c\gamma_1 - \gamma_2 + c\gamma_1}{1-c}\sqrt{-c} + \sqrt{K_1K_2}\cos(\theta+\phi) \\ \left(v_2 - \frac{b}{1-c}\right)\frac{1}{\sqrt{-c}} & = & \frac{\gamma_2 - c\gamma_2 - \gamma_2 + c\gamma_1}{1-c}\frac{1}{\sqrt{-c}} - \sqrt{K_1K_2}\cos(\theta+\phi) \end{array} \right. \quad \text{(A3)}$$

$$\left\{ \begin{array}{lcl} \left(v_1 - \frac{b}{1-c}\right)\sqrt{-c} & = & \frac{\gamma_1 - \gamma_2}{1-c}\left(\frac{1}{\sqrt{-c}}\right)^{-1} + \sqrt{K_1K_2}\cos(\theta+\phi) \\ \left(v_2 - \frac{b}{1-c}\right)\frac{1}{\sqrt{-c}} & = & c\frac{\gamma_1 - \gamma_2}{1-c}\frac{1}{\sqrt{-c}} - \sqrt{K_1K_2}\cos(\theta+\phi) \end{array} \right. \quad \text{(A4)}$$

$$\left\{ \begin{array}{lcl} \left(v_1 - \frac{b}{1-c}\right)\sqrt{-c} & = & (\gamma_1 - \gamma_2)\frac{\sqrt{-c}}{1-c} + \sqrt{K_1K_2}\cos(\theta+\phi) \\ \left(v_2 - \frac{b}{1-c}\right)\frac{1}{\sqrt{-c}} & = & -(\gamma_1 - \gamma_2)\frac{\sqrt{-c}}{1-c} - \sqrt{K_1K_2}\cos(\theta+\phi) \end{array} \right. \quad \text{(A5)}$$

Linearizing the system, we now obtain:

$$\left\{ \begin{array}{lclcl} v_1^* & = & \left(v_1 - \frac{b}{1-c}\right)\sqrt{-c} & = & (\gamma_1 - \gamma_2)\frac{\sqrt{-c}}{1-c} + \sqrt{K_1K_2}\cos\theta\sin\phi + \sqrt{K_1K_2}\sin\theta\cos\phi \\ v_2^* & = & \left(v_2 - \frac{b}{1-c}\right)\frac{1}{\sqrt{-c}} & = & -(\gamma_1 - \gamma_2)\frac{\sqrt{-c}}{1-c} - \sqrt{K_1K_2}\cos\theta\sin\phi - \sqrt{K_1K_2}\sin\theta\cos\phi \end{array} \right. \quad \text{(A6)}$$

$$\begin{pmatrix} | \\ v_1^* \\ | \\ v_2^* \\ | \end{pmatrix} = \begin{pmatrix} | & | & | \\ \frac{\sqrt{-c}}{1-c} & \sin\phi & \cos\phi \\ | & | & | \\ -\frac{\sqrt{-c}}{1-c} & -\sin\phi & -\cos\phi \\ | & | & | \end{pmatrix} \begin{pmatrix} \gamma_1 - \gamma_2 \\ \sqrt{K_1K_2}\cos\theta \\ \sqrt{K_1K_2}\sin\theta \end{pmatrix} \quad \text{(A7)}$$

APPENDIX B: ERROR PROPAGATION: SB2 CIRCULAR ORBIT

Let us adopt the following notations :

$$\Gamma = \gamma_1 - \gamma_2, \quad \text{(B1)}$$

$$C = \sqrt{K_1K_2}\cos\theta, \quad \text{(B2)}$$

$$S = \sqrt{K_1K_2}\sin\theta, \quad \text{(B3)}$$

$$c = -\frac{K_2}{K_1}, \quad \text{(B4)}$$

$$b = \gamma_2 - a\gamma_1. \quad \text{(B5)}$$

The above parameters correspond to the five independent variables adjusted by LOSP in the case of an SB2 circular system. From their best-fit values, one can deduced the orbital parameters of the two components as well as their related errors:

- $\gamma_1 = \frac{\Gamma+b}{1-c}$; $\quad \sigma^2_{\gamma_1} = \frac{1}{(1-c)^2}\sigma^2_\Gamma + \frac{1}{(1-c)^2}\sigma^2_b + \frac{(\Gamma+b)^2}{(1-c)^4}\sigma^2_c$
- $\gamma_2 = \frac{b+c\Gamma}{(1-c)}$; $\quad \sigma^2_{\gamma_2} = \frac{c^2}{(1-c)^2}\sigma^2_\Gamma + \frac{1}{(1-c)^2}\sigma^2_{\gamma_2} + \frac{\gamma_1^2}{(1-c)^2}\sigma^2_c$
- $K_1 = \sqrt{\frac{C^2+S^2}{-c}}$; $\quad \sigma^2_{K_1} = \frac{C^2K_1^2}{\left(S^2+C^2\right)^2}\sigma^2_C + \frac{S^2K_1^2}{\left(S^2+C^2\right)^2}\sigma^2_S + 2\frac{SC}{\left(S^2+C^2\right)^2}K_1^2\sigma^2_{CS} + \frac{K_1^2}{4a^2}\sigma^2_c$
- $K_2 = \sqrt{-c\left(S^2+C^2\right)}$; $\quad \sigma^2_{K_2} = \frac{K_2^2}{\left(S^2+C^2\right)^2}\left(S^2\sigma^2_S + C^2\sigma^2_C + 2CS\sigma^2_{CS}\right) + \frac{K_2^2}{4c^2}\sigma^2_c$
- $\theta = \arctan\left(\frac{S}{C}\right)$; $\quad \sigma^2_\theta = \frac{C^2}{\left(S^2+C^2\right)^2}\sigma^2_S + \frac{S^2}{\left(S^2+C^2\right)^2}\sigma^2_C - 2\frac{CS}{\left(S^2+C^2\right)^2}\sigma^2_{SC}$

- $T_0 = t_{arb} - \frac{P}{2\pi}\arctan\frac{S}{C}$; $\sigma^2_{T_0} = \frac{P}{2\pi}\sigma_\theta$
- $a_1 \sin i = \frac{K_1}{2\pi}P(\mathrm{sec})$; $\sigma_{a_1 \sin i} = \frac{P}{2\pi}\sigma_{K_1}$
- $a_2 \sin i = \frac{K_2}{2\pi}P(\mathrm{sec})$; $\sigma_{a_2 \sin i} = \frac{P}{2\pi}\sigma_{K_2}$
- $m_1 \sin^3 i = \frac{P}{2\pi G}\left(S^2 + C^2\right)^{3/2}(v-1)^2(-c)^{-1/2}$
- $m_2 \sin^3 i = \frac{P}{2\pi G}(1-c)^2\left(\frac{C^2+S^2}{-c}\right)^{3/2}$

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