Effects of “extra-mixing” processes on the periods of high-order gravity modes in main-sequence stars

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Abstract: In main-sequence stars, the chemical composition gradient that develops at the edge of the convective core is responsible for a non-uniform period spacing of high-order gravity modes. In this work we investigate, in the case of a 1.6 $M_\odot$ star, the effects on the period-spacing of extra mixing processes in the core (such as diffusion and overshooting).

1 Effects of overshooting and diffusion on the central μ-profile

We consider three models of a 1.6 $M_\odot$ star on the main sequence: a model without any “extra-mixing” process (model A), considering overshooting from the convective core (B) and including helium diffusion (C). Though the central hydrogen abundance is the same ($X_c = 0.3$), the models have a different chemical composition profile near the outer edge of the convective core (see Fig. 1). Taking A as the reference model, we see that overshooting (B) displaces the location of the μ-gradient, whereas diffusion (C) leads to a smoother chemical composition profile. As shown in the lower panel of Fig. 1, such differences are also reflected in the behaviour of the Brunt-Väisälä frequency N that, in its turn, determines the properties of gravity modes.

2 Effects on the period spacing

In white dwarfs it has been theoretically predicted and then observed (see for instance Metcalfe et al. (2003) and references therein) that the period spacing (hereafter ΔP) of g-modes is not constant, contrary to what is predicted by the first order asymptotic approximation of gravity modes (Tassoul 1980). This has been interpreted as the signature of sharp variations in N due to chemical composition gradients in the envelope and core of the star.

In analogy with the case of white dwarfs, in main-sequence models with a convective core we expect the formation of a *nonuniform period distribution*; this is in fact the case as presented in Fig. 2. The period spacing shows clear periodic components superposed to a constant ΔP expected for a model without sharp variations in N. The periodicity and amplitude of these

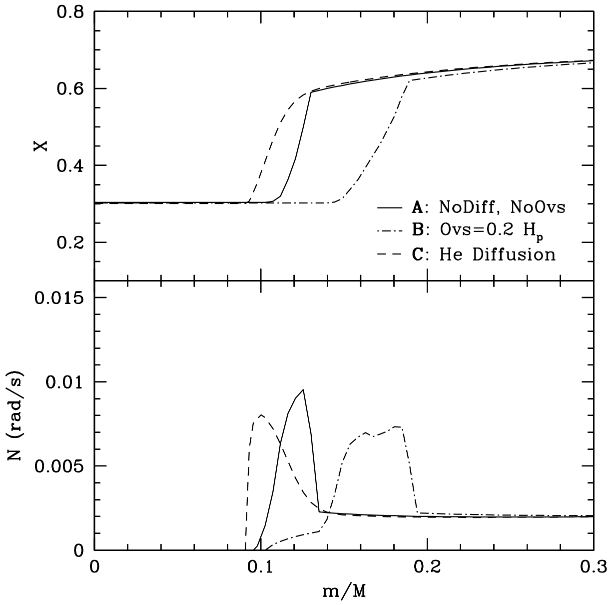

Figure 1: Behavior of the the hydrogen abundance profile (upper panel) and of the Brunt-Väisälä frequency (lower panel) in models of 1.6 $M_{\odot}$ with $X_c \simeq 0.3$. The different lines correspond to models calculated with no extra-mixing (A, continuous lines), overshooting (B, dashed-dotted) and helium diffusion (C, dashed). The different location and sharpness of the chemical composition gradient determines the behaviour of N (lower panel).

components can be related to the location and sharpness of the μ-gradient region by means of analytical expressions.

2.1 Analytical approximations

As described e.g. in Montgomery et al. (2003) the effect of a sharp feature in the model (a chemical composition gradient, for instance) can be estimated as the periodic component of the difference δP between the periods of the star showing such a sharp variation and the periods of an otherwise fictitious smooth model.

As a first example we model as a *step function* the sharp feature δN in the Brunt-Väisälä frequency located at a normalized radius $x = x_\mu$.

We then define

$$\Pi^{-1}(x) = \int_{x_0}^{x} \frac{|N|}{x'} dx' \ , \ \Pi_0^{-1} = \int_{x_0}^{1} \frac{|N|}{x'} dx' \ \text{ and } \ \Pi_\mu^{-1} = \int_{x_0}^{x_\mu} \frac{|N|}{x'} dx' \qquad (1)$$

where x_0 is the boundary of the convective core and we consider a model with a radiative envelope.

Following the approach of Montgomery et al. (2003), and using the asymptotic expression for g-modes periods P_k derived by Tassoul (1980):

$$P_k = \pi^2 \frac{\Pi_0}{L} (2k + n_e) \qquad (2)$$

where n_e is the effective polytropic index of the superficial layers, k is the radial order of the

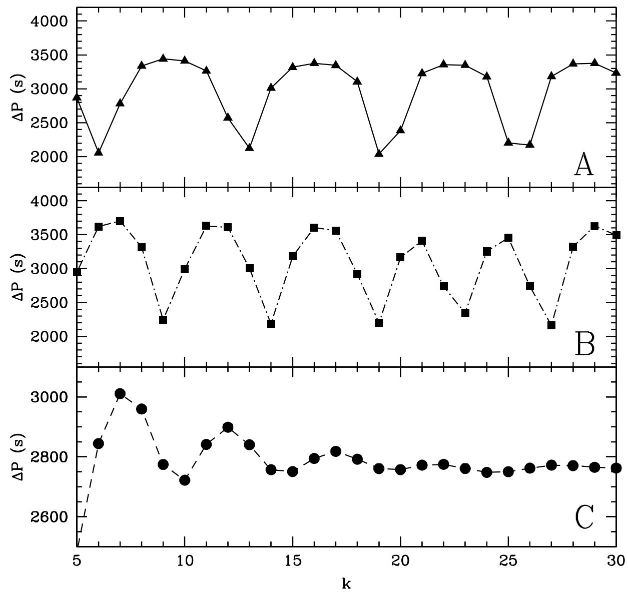

Figure 2: Period spacing $\Delta P = P_{k+1} - P_k$ as a function of the radial order k in $\ell = 1$ g-modes for the models presented in Figures 1 and 3. The periods of g modes were computed using the adiabatic stellar oscillation code OSC.

mode and $L = [\ell(\ell+1)]^{1/2}$, we find

$$\delta P_k \propto \frac{\Pi_0}{L} A \cos\left(2\pi \frac{\Pi_0}{\Pi_\mu} k + \phi\right), \tag{3}$$

where A is related to the sharpness of the variation in N.

From this simple approach we derive that the signature of a sharp feature in the Brunt-Väisälä frequency is a *periodic component in the periods of oscillations*, and therefore in the period spacing ΔP, whose periodicity in terms of the radial order k is given by

$$\Delta k \simeq \frac{\Pi_\mu}{\Pi_0} \tag{4}$$

and whose amplitude does not depend on the order k.

We now compare this approximation to the numerically computed period spacing. In Fig. 2 the periods (in terms of k) of the components are approximately 7 for model A and 5 for model B and C. Following Eq. (4) these periods should correspond to a location of the discontinuity (expressed as $\Pi_0/\Pi_\mu \simeq k^{-1}$) of 0.14 and 0.2: as shown in Fig. 3, these estimates accurately describe the locations of the sharp variation of N in the models.

The period spacing of the model computed with diffusion deserves, however, further inspection. As shown in the lower panel of Fig. 2, the amplitude of the components in the period spacing of model C, compared to model A and B, is considerably reduced. Moreover, the amplitude also becomes a decreasing function of the order k: this behaviour can be directly related to a smoother chemical composition profile.

In fact, the simple approach followed so far allows us to easily evaluate the effect of having a smoother variation in the Brunt-Väisälä frequency. Instead of modelling δN as a step function, we use a *ramp function* that, as shown in Fig. 3, better represents the variation of δN in model C.

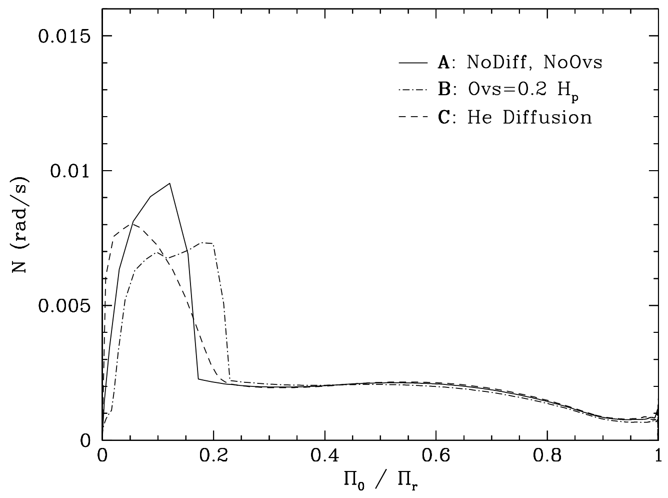

Figure 3: The Brunt-Väisälä frequency versus Π_μ/Π_0 for the models. Whereas the sharp variation in N in model A and B can be represented by a step function, in the case of model C (calculated with diffusion) it is better modelled by a ramp function.

In this case integration by parts leads to a sinusoidal component in δP_k whose *amplitude is modulated by a factor* $1/P_k$ and therefore decreases with increasing k, i.e.

$$\delta P_k \propto \frac{\Pi_0}{L}\frac{A'}{P_k}\cos\left(2\pi\frac{\Pi_0}{\Pi_\mu}k+\phi'\right) . \tag{5}$$

This simplified approach is therefore sufficient to account for the behaviour of ΔP (Fig. 2) in the model computed with diffusion, where the sharp feature in N described by a discontinuity not in N itself, but in its first derivative, generates a periodic component whose amplitude decreases with k.

3 Conclusions and prospects

In main-sequence stars, similarly to the case of white dwarfs, the deviations from a constant g-mode period spacing are sensitive probes of μ-gradients that develops at the outer edge of a convective core. These deviations can be interpreted by means of simple analytical expressions that could represent a possible seismic tool to study the detailed properties of chemical mixing in γ Doradus and SPB stars, where high-order gravity modes are observed.

The question whether such signatures could be detected given realistic observational errors and other uncertainties (e.g. effects of rotation on g-modes periods) needs however further investigation. A more detailed study will be presented in a forthcoming paper.

References

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