How can we probe the interiors of stars?

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Abstract: Twenty-five hundred years ago the Pythagorean Brotherhood invented the idea of the Music of the Spheres. That idea languished in scientific thought from the time of Kepler 400 years ago, until the 1970s when real sounds were found and recognised in the sun and stars. Stars pulsate both with sound waves in pressure modes and in gravity modes with buoyancy as the restoring force. Those pulsations allow us literally to see inside the stars to know their interiors. Oscillations in 1-D and 2-D are introduced here, leading to 3-D descriptions of the changing shapes of pulsating star. Some properties of pressure modes and gravity modes are explained, and a simple explanation of asteroseismology is given. Some selected cases illustrate amazing discoveries from our new ability to see inside the stars.

1 Introduction

1.1 The Music of the Spheres

Pythagoras of Samos (c. $569 - 475$ BC) is best-known now for the Pythagorean Theorem relating the sides of a right triangle: $a^2 + b^2 = c^2$, but his accomplishments go far beyond this. When Pythagoras was a young man (c. 530 BC) he emigrated to Kroton in southern Italy where he founded the Pythagorean Brotherhood who soon held secular power over not just Kroton, but more extended parts of Magna Grecia. He and his followers were natural philosophers (they invented the term "philosophy") trying to understand the world around them; in the modern sense we would call them scientists. They believed that there was a natural harmony to everything, that music, mathematics and what we now call physics were intimately related. In particular, they believed that the motions of the sun, moon, planets and stars generated musical sounds. They imagined that the Earth is a free-floating sphere and that the daily motion of the stars and the movement through the stars of the sun, moon and planets were the result of the spinning of crystalline spheres or wheels that carried these objects around the sky. The gods, and those who were more-than-human (such as Pythagoras), could hear the hum of the spinning crystalline spheres: they could hear the *Music of the Spheres* (see Koestler 1959).

The idea of the Music of the Spheres seems to resonate in the human mind; the expression is alive and current today, 2500 years later. A century after Pythagoras, Plato (c. $427 - 347$ BC) said that "a siren sits on each planet, who carols a most sweet song, agreeing to the motion of her own particular planet, but harmonising with all the others" (see Brewer 1894). Two millennia after Plato, Johannes Kepler ($1571 - 1630$) so believed in the Music of the Spheres that he spent years trying to understand the motions of the planets in terms of musical harmonies. He did admit that "no sounds are given forth," but still held "that the movements of the planets

are modulated according to harmonic proportions." It was only after Herculean efforts failed that Kepler gave up on what he wanted to be true, the Music of the Spheres, started over and discovered his famous third law for the planets, $P^2 = a^3$. It was this willingness to discard a cherished belief, an ancient and venerable idea, and begin again that made Kepler a truly modern scientist.

William Shakespeare $(1564 - 1616)$ was a contemporary of Kepler, and of course you can find the Music of the Spheres in Shakespeare (Merchant of Venice, v. 1):

> There's not the smallest orb which thou beholdest
> but in his motion like an angel sings
> Still quiring to the young-eyed cherubim

The Music of the Spheres never left artistic thought or disappeared from the language, but as a "scientific" idea it faded from view with Kepler's Laws of motion of the planets. And so it languished until the 1970s when astronomers discovered that there is resonant sound inside stars, that stars "ring" like giant bells, that there is a real Music of the Spheres.

1.2   Seeing with sound

In the opening paragraph of his now-classic book, *The Internal Constitution of the Stars* (Eddington 1926), Sir Arthur Stanley Eddington lamented:

> At first sight it would seem that the deep interior of the sun and stars is less accessible to scientific investigation than any other region of the universe. Our telescopes may probe farther and farther into the depths of space; but how can we ever obtain certain knowledge of that which is hidden behind substantial barriers? What appliance can pierce through the outer layers of a star and test the conditions within?

Eddington considered theory to be the proper answer to that question: From our knowledge of the basic laws of physics, and from the observable boundary conditions at the surface of a star, we can *calculate* its interior structure, and we can do so with confidence.

While we humans shower honours, fame and fortune on those who can run 100 m in less than 10 s, leap over a 2-m bar, or lift 400 kg over their heads, cheetahs, dolphins and elephants (if they could understand our enthusiasm for such competitions) would have a good laugh at us for those pitiful efforts. We are no competition for them in physical abilities. But *we* can calculate the inside of a star! That is at the zenith of human achievement. No other creature on planet Earth can aspire to this most amazing feat.

Some humility is called for, however. In *The Internal Constitution of the Stars* Eddington reminds us on page 1: "We should be unwise to trust scientific inference very far when it becomes divorced from opportunity for observational test." Indeed! Therefore he would have been amazed and delighted to know that there is now a way to see inside the stars – not just calculate their interiors – but literally *see*. We have invented Eddington's "appliance" to pierce the outer layers of a star: It is asteroseismology, the probing of stellar interiors through the study of their surface pulsations.

Stars are not quiet places. They are noisy; they have sound waves in them. Those sounds cannot get out of a star, of course; sound does not travel in a vacuum. But for many kinds of stars – the pulsating stars – the sound waves make the star periodically swell and contract, get hotter and cooler. With our telescopes we can see the effects of this: the periodic changes in

the star's brightness; the periodic motion of its surface moving up-and-down, back-and-forth. Thus we can detect the natural oscillations of the star and "hear" the sounds inside them.

Close your eyes and imagine that you are in a concert hall listening to an orchestra tuning up: The first violinist walks over to the piano and plunks middle-A which oscillates at 440 Hz. All the instruments of the orchestra then tune to that frequency. And yet, listen! You can hear the violin. You can hear the bassoon. You can hear the French Horn. You can hear the cello, the flute, the clarinet and the trumpet. Out of the cacophony you can hear each and every instrument separately and identify them, even though they are all playing exactly the same frequency. How *do* you do that?

Each instrument in the orchestra is shaped to put power into some of its natural harmonics and to damp others. The shape of the instrument determines its natural oscillation modes, so determines which harmonics are driven and which are damped. It is the combination of the frequencies, amplitudes and phases of the harmonics that defines the character of the sound emanated, that gives the timbre of the instrument, that gives it its unique sound. It is the combination of the harmonics that defines the rate of change of pressure with time emanating from the instrument – that defines the sound waves it creates.

A sound wave is a pressure wave. In a gas this is a rarefaction and compression of the gas that propagates at the speed of sound. The high pressure pushes, compresses and propagates. Ultimately, this is done at the molecular level; the information that the high pressure is coming is transmitted by individual molecular collisions. In the adiabatic case, the speed of sound is $v_s = \sqrt{\frac{\Gamma P}{\rho}}$, where Γ is the adiabatic exponent, P is pressure and ρ is density. Of course, for a perfect gas $P = \frac{\rho k T}{\mu}$, where μ is mean molecular weight, thus $v_s = \sqrt{\frac{\Gamma k T}{\mu}}$. The changes in pressure are therefore accompanied by changes in density and temperature. Principally, as we can see from the last relationship, the speed of sound depends on the temperature and chemical composition of the gas. Thus, if the temperature is higher, and the molecules are moving more quickly, then they collide more often and the sound speed is higher. And at a given temperature in thermal equilibrium, lighter gases move more quickly, collide more often, and the sound speed is higher than for heavier gases.

This last effect is the cause of a well-known party trick. Untie a helium balloon, breathe in a lung-full of helium, and you will sound like Donald Duck when you talk! The speed of sound in helium at standard temperature and pressure is $970 \, \mathrm{m \, s^{-1}}$, compared to $330 \, \mathrm{m \, s^{-1}}$ in air (78% N_2, 21% O_2 and 1% Ar). With the nearly three times higher sound speed in helium the frequency of your voice goes up by that factor of three, hence the high-pitched hilarity. (As an aside: breathing helium is safe, so long you do not do it for too long, i.e. so long as it is not the only thing you are breathing. It is inert and will not react chemically. Deep-sea divers breathe heliox, a mixture of helium and oxygen, to reduce decompression time compared to breathing an air mixture, since helium comes out of solution in the blood more quickly than does molecular nitrogen.)

Thus, if you can measure the speed of sound in a gas, you have information about the temperature and chemical composition of that gas, and, from the equation of state, the pressure and density. Stars are made of gas, and they are like giant musical instruments. They have natural overtones (not the harmonics of musical instruments, so the sounds of the stars are dissonant to our ears when we play them at audible frequencies), and just as you can hear what instrument makes the sounds of an orchestra, i.e. you can "hear" the shape of the instrument, we can use the frequencies, amplitudes and phases of the sound waves that we detect in the stars to "see" their interiors – to see their internal "shapes". A goal of asteroseismology is to measure the sound speed throughout a star so that we can know those fundamental parameters of the stellar structure.

We humans are incredibly *visual* creatures; for us, sight is a dominant sense. We think "seeing is believing". Yet other animals perceive the world in other ways. Take a dog for a walk. The dog dedicates 60 times more brain to its sense of smell than you do. Dogs can see, but for them "smelling is believing". If a dog sees an object that it does not understand and does not trust, it will approach cautiously (sometimes with its hackles up) until the suspicious object can be smelled, and *then* the situation will be clarified and the dog will "know" the object. For them "smelling is believing."

What happens to you when you "see"? Does your brain detect the light? Is there a real image in your head? Of course not. Your eye forms an image on your retina, the photons are absorbed, an electro-chemical signal passes down your optic nerve to the part of your brain that interprets the incoming visual signal, and you have the impression that there is a 3-D theatre in your head. You "see" an image of the world.

So what then happens to you when you "hear"? Does your brain hear the sound? Are the sound waves in your head? Again, of course not. Your eardrum oscillates in and out with the increasing and decreasing pressure of the sound wave. Through the bones in your ears and through sensitive hairs the sound is transmitted, then transformed into an electro-chemical signal that passes to the part of your brain that interprets the incoming aural signal, and you have the impression that there is a 3-D sound system in your head. You "hear" the world.

While our perceptions of sight and sound are very different experiences, they are physiologically similar, and they are both providing us with information about the world around us. So is possible to "see" with sound? Yes. Of course it is. Bats do it with echo-locating. They emit sounds and the returning echoes tell the bat where everything in its environment is, down to the small insects that they catch for food (and also provide velocity information from the Doppler shift). Those sounds are converted to electro-chemical signals in the bat's brain, and the bat has a picture of the world around it. That is "seeing" with sound. A colony of a million bats leaving a narrow cave mouth in the dark has few collisions; the bats can "see" each other. It is not possible to get inside the mind of another creature. We cannot even do it with a fellow human; we cannot know if another person has the same experience that we have, e.g., of colour, of tone, of taste. We assume that they do, and get along well with that assumption, so similarly we may assume that bats "see" the world through sound. Their sense of hearing powers the 3-D theatre in their minds, just as our sense of sight does for us. We may surmise that the experiences of seeing with light or sound are similar.

Similarly, asteroseismology uses astronomical observations – photometry and spectroscopy – to extract the frequencies, amplitudes and phases of the sounds at a star's surface. Then we use basic physics and mathematical models to know the sound speed inside a star and from that to determine its temperature throughout its interior. With reasonable assumptions about chemical composition and knowledge of appropriate equations of state, pressure and density can be derived. These are, in a real sense, all the equivalent of the electro-chemical signals in our brains. We build up a picture in the 3-D theatre in our minds of what the inside of a star looks like. We see inside the star. The sounds tell us what the interior structure of the star *has* to be.

Who hasn't been amazed to see a picture of the face of a foetus in the womb, imaged using ultrasound waves? Do you question the reality of that? No. That is a real picture of the baby before it is born. Identically, using infrasound from the stars, the pictures of their insides that we see using asteroseismology have this same reality.

We have answered Eddington's question, "What appliance can pierce through the outer layers of a star and test the conditions within?"

The answer is: **Asteroseismology, the real Music of the Spheres.**

1.3 Can we "hear" the stars?

So you have been persuaded that there are sounds in stars and we can use those to "see" inside them. But can we actually hear them? Is there really a Music of the Spheres? Amazingly, the answer to that is also yes.

What we consider to be musical is mostly the relationships among the frequencies, amplitudes and phases of sounds, not their absolute pitch. A few humans have perfect pitch, and serious musicians and music-lovers do care about the key that a piece of music is played in – for the sound, and sometimes for the ease of playing it. But for most people a change of key does not change the character of the music – a melody is still recognisable in another key – because the *relationships* among the frequencies are not changed.

Now think about this: We have sound recording equipment that can detect the ultrasound of bats. We record the frequencies, amplitudes and phases of those sounds. Then, we simply perform a key change and shift the frequencies down into the audible range while keeping the frequency ratios the same, while keeping the amplitude and phase relationships; i.e. we perform a change of key. Played through a speaker we can then hear what bats sound like. It is a legitimate experience and may even be close to what it would be like to have ultrasound hearing and actually hear the bats directly with our own ears. (Fortunately, we cannot hear the bats, for they are loud and they are noisy; we probably would not like it.)

Similarly, with the right equipment we may record the infrasounds of whales, perform a key change to shift them up in frequency into the audible, and experience the haunting "songs" of the whales. This, too, is really hearing the sounds of the whales. (Unfortunately, the whales can hear the infrasounds of our many ships, so their environment has become vastly noisier over the last two centuries.)

Therefore, it is fair to say that when we observe the frequencies, amplitudes and phases of a pulsating star that are caused by sounds in the star, and we shift those with a key change up into the audible and play them through a speaker, we are experiencing the real Music of the Spheres. Pythagoras and Kepler would have been amazed.

1.3.1 StellarMusicNo1

While it is possible to use our observations of pulsating stars to generate sound files for the stars, and listen to them, we do not do science that way. Asteroseismology uses the frequencies, amplitudes and phases from observations of pulsating stars directly to model and probe the stellar interiors. But the sounds are intellectually intriguing, and they are even aesthetically pleasing.

The first musical composition based on the sounds of the stars is called StellarMusicNo1, by Jenõ Keuler and Zoltán Kolláth of Konkoly Observatory. Discussion of the music, a sound file and a score can be found on Zoltán Kolláth's website[1].

1.4 Pressure modes and gravity modes

When an idea is being discussed in Belgium, the response often begins, "Well, it's not as simple as that!" This expression, much loved by Belgian astronomers, is often useful to the rest of us, too. Therefore, given all that has been said so far: It is not as simple as that.

There is more to stellar pulsation than acoustic waves – sound waves – in stars. Those acoustic waves are known as "pressure" modes, or p-modes. There are equally important "gravity" modes, or g-modes, where the restoring force of the pulsation is not pressure, but

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[1]https://www.konkoly.hu/staff/kollath/stellarmusic/

buoyancy. Much of the picture of stellar pulsation that we have been painting is a valid view of gravity modes, too; they also probe the interiors of stars, and let us see below their surfaces. But gravity modes are not acoustic – they are not caused by sounds in the stars. We will discuss these two kinds of pulsation in parallel as our view of stellar pulsation grows clearer.

Now we need to build in our minds a picture of what the 3-D pulsations of stars look like.

2   1-D oscillations

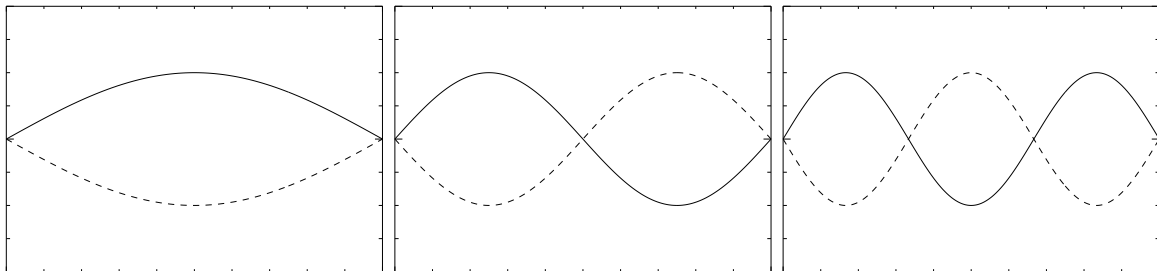## 2.1   1-D oscillations on a string

Figure 1: The first three oscillation modes for a string that is fixed at both ends, such as a violin string or a guitar string. On the left is the fundamental mode; in the centre is the first overtone which has a single node; and on the right is the second overtone which has two nodes. Note that the nodes are uniformly spaced.

Fig. 1 shows the fundamental mode and the first and second overtone modes for a vibrating string such as those on violins, guitars or any musical string instrument. The frequencies of these modes depend on the length of the string, the tension and the material the string is made of. Importantly, the tension and composition of the string are uniform along its length. Under those conditions the first overtone mode has twice the frequency of the fundamental mode, the second overtone mode has a frequency three times that of the fundamental mode, and so on. We therefore refer to these overtones as "harmonics", since they have small integer ratios. To our ears the frequencies with small integer ratios, such as 2:1, 3:2, 4:3, are harmonious. But note that here we distinguish the words "overtone" and "harmonic"; while they are the same for modes on a uniform string, they are not the same for stars, as we will see.

2.2   1-D oscillations in an organ pipe

If instead of a string we think of the oscillations of the air in an organ pipe, or any wind instrument with one closed end, then there is a displacement node at the closed end of the pipe, and the other, open end has a displacement antinode. Fig. 2 shows this schematically. As for the string in the last section, note that the overtones are harmonic with small integer ratios – in the cases in Fig. 2 these are 3:1 and 5:1 – since the air temperature and chemical composition are uniform within the pipe, so the sound speed is constant along the pipe. While the organ pipe is in some ways a simple analogue of a radially pulsating star, the uniform temperature is far from true for stars, as we will see, and therein lies a big difference.
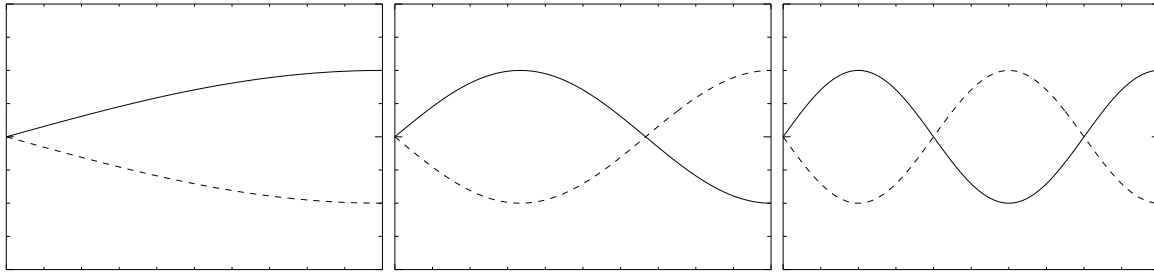
Figure 2: The first three oscillation modes for an organ pipe with one end (on the left) closed, and one end (on the right) open. On the left is the fundamental mode; in the centre is the first overtone which has a single node; and on the right is the second overtone which has two nodes. Note that the open end is an anti-node in the displacement of the air, and that the nodes are uniformly spaced.

3   2-D oscillations in a drum head

To imagine the oscillations of a 2-D membrane, a drumhead is easy to visualise, as can be seen in Fig. 3. Because the drumhead is two-dimensional, there are nodes in two orthogonal directions. One set of modes has nodes that are concentric circles on the drumhead, and those modes are called *radial* modes. For a drumhead the rim is always a node, so the fundamental radial mode simply has the drumhead move up and down with circular symmetry with maximum amplitude at the centre, which is an antinode. The first radial overtone has a node that is a circle on the drumhead with the centre and outside annulus moving in antiphase; the second radial overtone has two concentric circles as nodes, and so on. (These radial modes are rapidly damped in an actual drumhead, so contribute only to the initial sound of the drum being struck, and not much to the ringing oscillations that follow.)

The second direction of nodes in a drumhead gives rise to the *nonradial* modes. The first nonradial mode is the dipole mode which has a node that is a line across the drumhead dividing it in two, so that the two halves oscillate in antiphase. The second nonradial overtone has two crossing nodes dividing the drum into four equal sections. Of course, there are modes that have both radial and nonradial nodes. The important point about the drumhead is that these modes do *not* have frequencies with small integer ratios, so the drum is not harmonic; it does not ring with a musical sound[2]. For a uniform density and tension drumhead, the solutions to the oscillation equations are Bessel functions; the radial nodes in Fig. 3 are schematic only. To visualise drumhead oscillations better, excellent graphical movies can be found on the web site of Dan Russell[3].

4   3-D oscillations in stars

Stars are three-dimensional, so their natural oscillation modes have nodes in three orthogonal directions. Those are concentric radial shells (r), lines of latitude (θ) and lines of longitude (φ). For a spherically symmetric star the solutions to the equations of motion have displacements in the (r, θ, φ) directions and are given by

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[2]Tympani do have a musical tone. This is the result of careful design where the air pressure in the drum damps some modes, and allows those that are close to harmonic to oscillate, thus giving a recognisable note.

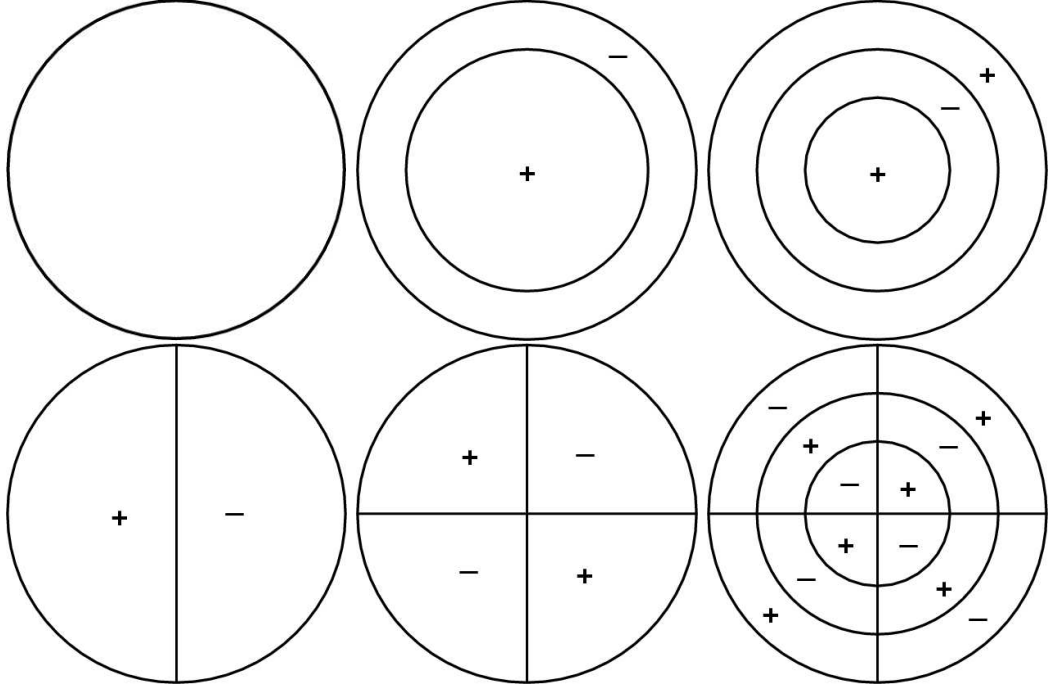[3]http://www.kettering.edu/ drussell/Demos/MembraneCircle/Circle.html

Figure 3: Schematic representations of some oscillation modes in a drum head. The rim of the drum is fixed, so is forced to be a node in all cases. The top left circle represents the fundamental radial mode for the drum: the rim is a node and the centre of the drum is an anti-node. The middle top figure represents the first *radial* overtone, with one node which is a concentric circle. The plus and minus signs indicate that the outer annulus moves outwards while the inner circle moves inwards, and vice versa. The top right figure represents the second radial overtone. The bottom left figure shows the simplest *nonradial* mode for a drum, the dipole mode, where a line across the middle of the drum is a node and one side moves up, while the other moves down, then vice versa. The middle bottom panel in the quadrupole nonradial mode, and the bottom right figure shows the second overtone quadrupole mode. The modes are characterised by quantum numbers, one for the number of radial nodes, and one for the number of nonradial nodes. So reading from left-to-right, top-to-bottom, the modes are numbered (0,0), (1,0), (2,0), (0,1), (0,2) and (2,2). A similar notation in 3-D exists for stellar pulsation modes, as we will see.

$$\xi_r\left(r,\theta,\varphi,t\right) = a\left(r\right) Y_\ell^m\left(\theta,\varphi\right) \exp\left(i\omega t\right) \tag{1}$$

$$\xi_\theta\left(r,\theta,\varphi,t\right) = b\left(r\right) \frac{\partial Y_\ell^m\left(\theta,\varphi\right)}{\partial\theta} \exp\left(i\omega t\right) \tag{2}$$

$$\xi_\varphi\left(r,\theta,\varphi,t\right) = \frac{b\left(r\right)}{\sin\theta} \frac{\partial Y_\ell^m\left(\theta,\varphi\right)}{\partial\varphi} \exp\left(i\omega t\right) \tag{3}$$

where ξ_r, ξ_θ and ξ_φ are the displacements, $a(r)$ and $b(r)$ are amplitudes, ω is the oscillation frequency and $Y_\ell^m\left(\theta,\varphi\right)$ are spherical harmonics given by

$$Y_\ell^m\left(\theta,\varphi\right) = \sqrt{\frac{2\ell+1}{4\pi}\frac{\left(\ell-m\right)!}{\left(\ell+m\right)!}} P_\ell^m\left(\cos\theta\right) \exp\left(im\varphi\right) \tag{4}$$

and $P_\ell^m (\cos \theta)$ are Legendre polynomials given by

$$P_\ell^m (\cos \theta) = \frac{(-1)^m}{2^\ell \ell!} \left(1 - \cos^2 \theta \right)^{\frac{m}{2}} \frac{\mathrm{d}^{\ell+m}}{\mathrm{d} \cos^{\ell+m} \theta} \left(\cos^2 \theta - 1 \right)^\ell \tag{5}$$

where θ is measured from the pulsation pole, the axis of symmetry. In most pulsating stars that axis coincides with the rotation axis. The main exceptions are the rapidly oscillating Ap stars where the axis of pulsational symmetry is the magnetic axis which is inclined to the rotational axis (see Section 7.3 below).

As with the drumheads, where there were two quantum numbers to specify the modes, for 3-D stars there are three quantum numbers to specify these modes: n is the number of radial nodes and is called the *overtone* of the mode[4]; ℓ is the *degree* of the mode and specifies the number of surface nodes that are present; m is the *azimuthal order* of the mode and specifies how many of the surface nodes are lines of longitude. It follows therefore that the number of surface nodes that are lines of latitude is equal to $\ell - m$. The values of m range from $-\ell$ to $+\ell$, so there are $2\ell + 1$ m-modes for each degree ℓ.

What do these modes in stars look like?

4.1   Radial modes

The simplest modes are the *radial* modes with $\ell = 0$, and the simplest of those is the fundamental radial mode with $n = 0$. In this mode the star swells and contracts, heats and cools, spherically symmetrically with the core as a node and the surface as a displacement antinode. It is the 3-D analogy to the organ pipe in its fundamental mode shown in the left-hand panel of Fig. 2. This is the usual mode of pulsation for Cepheid variables and for RR Lyrae stars, amongst others.

The first overtone radial mode has $n = 1$ with a radial node that is a concentric shell within the star. As we are thinking in terms of the radial displacement, that shell is a node that does not move; the motions above and below the node move in antiphase. As an example, in the roAp stars (which are nonradial pulsators) radial nodes can be directly observed in their atmospheres with just this kind of motion in antiphase above and below the radial node (Kurtz, Elkin & Mathys 2005). The surface of the star is again an antinode.

There are Cepheid variables, RR Lyrae stars and δ Scuti stars that pulsate simultaneously in the fundamental and first overtone radial modes. In the cases of the Cepheids and RR Lyrae stars they are known as double-mode Cepheids and RRd stars, respectively. For the Cepheids the ratio of the first overtone period to the fundamental period is 0.71; for the δ Scuti stars it is 0.77. This is in obvious contrast with the 0.33 ratio found in organ pipes and the 0.5 ratio found on strings (see Figs 1 and 2).

This difference is profound and it is our first use of asteroseismology. If the star were of uniform temperature and chemical composition (so that the sound speed were constant), then the ratio would be similar to that in the organ pipe. The larger ratios in the Cepheids and δ Scuti stars is a direct consequence of the sound speed gradient in them, hence the temperature and (in places) chemical composition gradients. The small, but significant differences between the Cepheid and δ Scuti ratios is a consequence of the Cepheid giant star being more centrally condensed than the hydrogen core-burning δ Scuti star. Thus, just by observing two pulsation frequencies we have had our first look into the interiors of some stars.

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[4]Sometimes k is preferred to represent this quantum number, particularly amongst those working on pulsating white dwarf stars.

4.2   Nonradial modes

The simplest of the nonradial modes is the axisymmetric *dipole* mode with $\ell = 1, m = 0$. For this mode the equator is a node; the northern hemisphere swells up while the southern hemisphere contracts, then vice versa; one hemisphere heats while the other cools, and vice versa – all with the simple cosine dependence of $P_1^0 (\cos \theta) = \cos \theta$, where θ is the latitude. There is no change to the circular cross-section of the star, so from the observer's point of view, the star seems to oscillate up and down in space.

That is disturbing to contemplate. What about Newton's laws? How can a star "bounce" up and down in free space without an external driving force? The answer is that an incompressible sphere cannot do this; it cannot pulsate in a dipole mode. After a large earthquake the Earth oscillates in modes such as those we are describing. But it does not oscillate in the dipole mode and bounce up and down in space. It cannot. There was a time when it was thought that stars could not do this either (Pekeris 1938), but first Smeyers (1966) in the adiabatic case, then Christensen-Dalsgaard (1976) more generally showed that the centre-of-mass of a star is not displaced during dipole oscillations, so stars can pulsate in such modes.

Nonradial modes only occur for $n \geq 1$, so in the case of the $\ell = 0$ dipole mode, there is at least one radial node within the star. While the outer shell is displaced upwards from the point of view of the observer, the inner shell is displaced downwards and the centre of mass stays fixed. Dipole modes are the dominant modes observed in the rapidly oscillating Ap stars, and are also seen in many other kinds of pulsating variables.

Modes with two surface nodes ($\ell = 2$) are known as quadrupole modes. For the $\ell = 2$, $m = 0$ mode the nodes lie at latitudes $\pm 35°$, since $P_2^0 (\cos \theta) = \frac{3}{2} \cos \theta - \frac{1}{2}$. The poles swell up (and heat up, although not usually in phase with the swelling) while the equator contracts (and cools), and vice versa. Fig. 4 shows and explains a set of *octupole* modes with $\ell = 3$, giving a mental picture of what the modes look like.

4.3   The effect of rotation: The m-modes

In eqs. 1 and 4 it can be seen that for *m-modes* (i.e., modes with $m \neq 0$) the exponentials in the two equations combine to give a time dependence that goes as $\exp[i(\omega t + m\varphi)]$. This phase factor in the time dependence means that the m-modes are travelling waves, where our sign convention is that modes with positive m are travelling against the direction of rotation (*retrograde* modes), and modes with negative m are travelling in the direction of rotation (*prograde* modes).

For a spherically symmetric star the frequencies of all $2\ell + 1$ members of a multiplet (such as the octupole septuplet $\ell = 3, m = -3, -2, -1, 0, +1, +2, +3$) are the same. But deviations from spherical symmetry can lift this frequency degeneracy, and the most important physical cause of a star's departure from spherical symmetry is rotation. In a rotating star the Coriolis force causes pulsational variations that would have been up-and-down to become circular with the direction of the Coriolis force being against the direction of rotation. Therefore, the prograde m-modes travelling in the direction of rotation have frequencies slightly lower than the $m = 0$ axisymmetric mode, and the retrograde modes going against the rotation have slightly higher frequencies, *in the co-rotating reference frame* of the star, thus the degeneracy of the frequencies of the multiplet is lifted.

This was discussed by Ledoux (1951) in a study of the β Cep star β CMa. In the *observer's frame of reference* the Ledoux rotational splitting relation for a uniformly rotating star is

$$\sigma_{n\ell m} = \sigma_{n\ell 0} - m \left(1 - C_{n\ell}\right) \Omega \qquad (6)$$
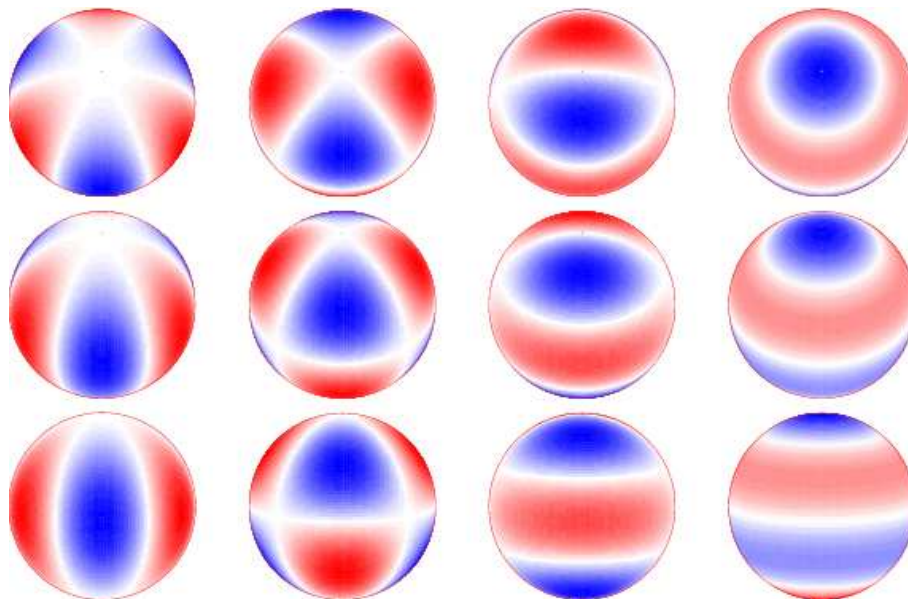
Figure 4: The $\ell = 3$ octupole modes. The rows show the modes from different viewing angles; the top row is for an inclination of the pulsation pole of $30°$, the second row is for $60°$, and the bottom row is for $90°$. The white bands represent the positions of the surface nodes; in the on-line colour version of the diagram red and blue represent sections of the star that are moving in (out) and/or heating (cooling), then vice versa. In the black-and-white printed version alternating dark sections are moving in opposite directions at any given time. The right-hand column shows the axisymmetric octupole mode ($\ell = 3, m = 0$) where the nodes lie at latitudes $\pm51°$ and $0°$. The second column from the right shows the *tesseral* (meaning $0 < m < \ell$) $\ell = 3, m = \pm1$ mode with two nodes that are lines of latitude and one that is a line of longitude. The third column from the right is the tesseral $\ell = 3, m = \pm2$ mode, and the left column shows the *sectoral* mode with $\ell = 3, m = \pm3$. Importantly, rotation distinguishes the sign of m, as discussed in the Section 4.3 Figure courtesy of Conny Aerts.

where $\sigma_{n\ell m}$ is the observed frequency, $\sigma_{n\ell 0}$ is the unperturbed central frequency of the multiplet (for which $m = 0$) which is unaffected by the rotation, $C_{n\ell}$ is a small constant, and Ω is the rotation frequency. If we rewrite eq. 6 as

$$\sigma_{n\ell m} = \sigma_{n\ell 0} + mC_{n\ell}\Omega - m\Omega, \tag{7}$$

then it is easy to see that the Coriolis force reduces the frequency of the $-m$ prograde mode slightly in the co-rotating rest frame, but then the rotation frequency is added to that since the mode is going in the direction of rotation. Likewise the $+m$ retrograde mode is travelling against the rotation so has its frequency is the observed frame reduced by the rotation frequency.

In the end we end up with a multiplet with $2\ell + 1$ components all separated by $(1 - C_{n\ell})\Omega$. In a real star the various components of the multiplet may be excited to different amplitudes, and some may not have any observable amplitude, so all members of the multiplet may not be present. The importance for asteroseismology is that where such rotationally-split multiplets are observed, the ℓ and m for the modes may be identified and the splitting used to measure the rotation rate of the star. Where multiplets of modes of different degree are observed, it is possible to gain knowledge of the *interior* rotation rate of the star – something that is not knowable by any other means.

In the case of the sun, helioseismology has spectacularly measured the differential rotation rate of the sun down to about half way to the core. Below the convection zone at $r/R_\odot \sim 0.7$ the sun rotates approximately rigidly with a period close to the 27-d period seen at latitudes of about 35° on the surface (see Thompson et al. 2003). Within the convection zone the rotation is not simply dependent on distance from the solar rotation axis, as had been expected in the absence of any direct observation. It is a remarkable triumph of helioseismology that we can know the internal rotation behaviour of the sun – thanks to m-modes!

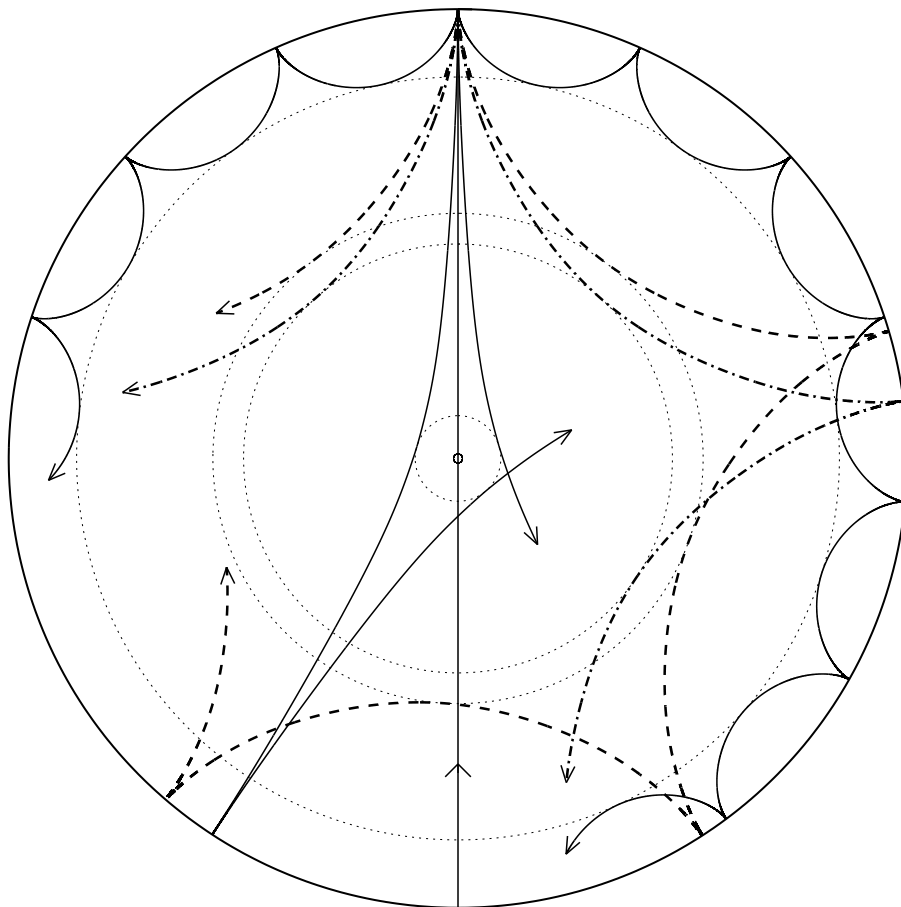## 4.4   So how does asteroseismology work?

Figure 5: This diagram shows the cross-section of a star with the ray paths of a selection of p-modes of different degree ℓ. The dotted circles represent the lower boundaries of the *acoustic cavities* for each mode. Figure courtesy of Jørgen Christensen-Dalsgaard.

Since p-modes are acoustic waves, for modes that are not directed at the centre of the star (i.e. the nonradial modes) the lower part of the wave is in a higher temperature environment than the upper part of the wave, thus in a region of higher sound speed. As a consequence the wave is refracted back to the surface, where it is then reflected, since the acoustic energy is trapped in the star, as can be seen in Fig. 5. While the number of reflection points is not equal to the degree of the mode, higher ℓ modes have more reflection points. This means that high degree modes penetrate only to a shallow depth, while lower degree modes penetrate more deeply. The frequency of the mode observed at the surface depends on the sound travel

time along its ray path, hence on the integral of the sound speed within its "acoustic cavity". Clearly, if many modes that penetrate to all possible depths can be observed on the surface, then it is possible to "invert" the observations to make a map of the sound speed throughout the star, and from that deduce temperature profile, with reasonable assumptions about the chemical composition. In the sun the sound speed is now known to a few parts per thousand over 90% of its radius. To do the same for other stars is the ultimate goal of asteroseismology.

A review of the theory of helioseismology can be found, for example, in Christensen-Dalsgaard (2002), and rigorous discussions of the theory of asteroseismology can be found in the course notes of Conny Aerts, Jørgen Christensen-Dalsgaard and others which can be found through links on the website of the European Network of Excellence in Asteroseismology, ENEAS[5].

Thus asteroseismology lets us literally see the insides of stars because different modes penetrate to different depths in the star. But as was noted in Section 1.4, it is not so simple as just p-modes. We can also see inside the stars with g-modes. In fact, for some stars, and for parts of others, we can only see with g-modes.

4.5   p-modes and g-modes

There are two main sets of solutions to the equations of motion for a pulsating star, and these lead to two types of pulsation modes: p-modes and g-modes. For the p-modes, or pressure modes, pressure is the primary restoring force for a star perturbed from equilibrium. These p-modes are acoustic waves and have gas motions that are primarily vertical. For the g-modes, or gravity modes, buoyancy is the restoring force and the gas motions are primarily horizontal. There is also an f-mode that is somewhere in-between the p-modes and g-modes and only exists for the lowest overtone and $\ell \geq 2$.

There are three other important properties of p-modes and g-modes: 1) as the number of radial nodes increases the frequencies of the p-modes increase, but the frequencies of the g-modes decrease, as is shown in Fig. 6; 2) the p-modes are most sensitive to conditions in the outer part of the star, whereas g-modes are most sensitive to the core conditions, as is shown in Fig. 7; 3) for $n >> \ell$ there is an asymptotic relation for p-modes which are approximately equally spaced in frequency, and there is another asymptotic relation for g-modes which are approximately equally spaced in period.

The asymptotic relations are very important in many pulsating stars. From Tassoul (1980, 1990) they are for the p-modes:

$$\nu_{n\ell} = \Delta\nu_0 \left(n + \frac{\ell}{2} + \epsilon \right) + \delta\nu \tag{8}$$

where n and ℓ are the overtone and degree of the mode, ϵ is a constant of order unity, $\delta\nu$ is known as the "small separation"; $\nu_{n\ell}$ is the observed frequency, $\Delta\nu_0$ is known as the "large separation" and is the inverse of the sound travel time for a sound wave from the surface of the star to the core and back again, given by

$$\Delta\nu_0 = \left(2 \int_0^R \frac{\mathrm{d}r}{c(r)} \right)^{-1} \tag{9}$$

where $c(r)$ is the sound speed. The large separation is obviously sensitive to the radius of the star, hence near the main sequence it is a good measure of the mass of the star. The small
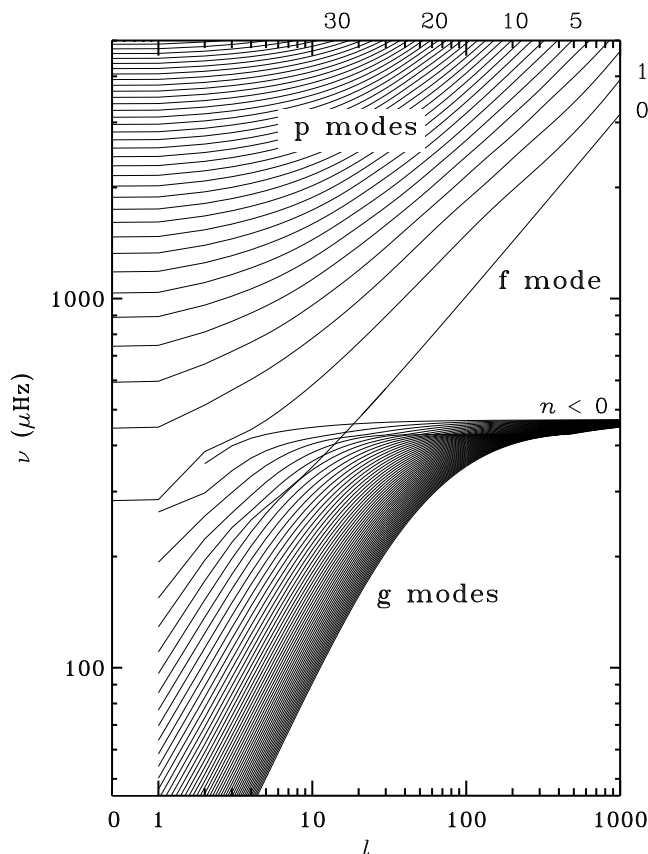
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[5]http://www.eneas.info/

Figure 6: This diagram plots the degree ℓ of a mode versus its frequency for a solar model. It clearly illustrates the general property of p-modes that frequency increases with overtone n and degree ℓ. For g-modes frequency decreases with higher overtone, but increases with n if we use the convention that n is negative for g-modes. Frequency still increases with degree ℓ for g-modes, just as it does for p-modes. Some values of the overtone n are given for the p-modes lines in the upper right of the figure. Note that while continuous lines are shown, the individual modes are discrete points that are not resolved here. Figure courtesy of Jørgen Christensen-Dalsgaard.

separation is sensitive to the core condensation, hence age of the star.

For the g-modes the nearly uniform period spacing is given by

$$\Pi_{n\ell} = \frac{\Pi_0}{\sqrt{\ell\,(\ell+1)}}\,(n+\epsilon) \tag{10}$$

where n and ℓ are again the overtone and degree of the mode, ϵ is a small constant, and Π_0 is given by

$$\Pi_0 = 2\pi^2 \left(\int \frac{N}{r}\mathrm{d}r \right)^{-1} \tag{11}$$

where N is the Brunt-Väisälä frequency and the integral is over the cavity in which the g-mode propagates (as in the right panel of Fig. 7). Deviations of the period spacing for g-modes are used to diagnose stratification in stars, since strong mean molecular weight gradients trap modes and cause deviations from the simple asymptotic relation given in eq. 10. This technique
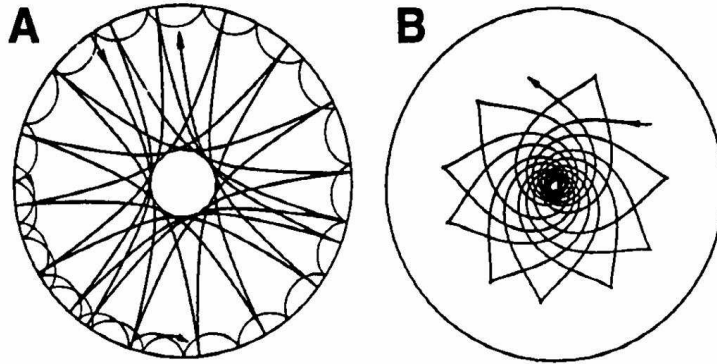
Figure 7: This diagram shows ray paths for two p-modes on the left and one g-mode on the right. The higher degree p-mode has $n = 8$, $\ell = 100$; the lower degree p-mode has $n = 8$, $\ell = 2$. The g-mode has $n = 10$, $\ell = 5$. Note that the g-mode is, in this case, trapped in the interior, since this is for a solar model and the g-modes do not propagate in the convective out part of the sun, although they potentially may very weakly may be observable at the surface from their effects on the convection zone. This figure illustrates that the g-modes are sensitive to the conditions in the very core of the star, an important property. From Gough et al. (1996); used by permission.

has been particularly successful in measuring the stratification in white dwarf atmospheres with carbon-oxygen cores and layers of helium and hydrogen above (see Section 7.4).

5   An asteroseismic HR diagram for p-mode pulsators

Fig. 8 shows a power spectrum of the radial velocity variations observed over a time span of $9.5\,\mathrm{yr}$ for the sun by BiSON, the Birmingham Solar Oscillation Network[6]. This shows the "comb" of frequencies expected from eq. 8 for high overtone, low degree ($n >> \ell$) p-modes. The noise level is so stunningly low in this diagram that it is essentially invisible at this scale. It is equivalent to an amplitude of only $0.5\,\mathrm{mm\,s^{-1}}$, precise enough to detect a mode with a total displacement over the whole pulsation cycle of only 10s of cm! It is noteworthy that the comb of frequencies consists of alternating even and odd ℓ-modes, as expected from eq. 8, where it can be seen that to first order modes of (n, ℓ) and $(n - 1, \ell + 2)$ have the same frequency. It is the small separation, $\delta\nu$, that lifts this degeneracy.

That may be seen in Fig. 9 which is a portion of an amplitude spectrum of the radial velocity variations of the sun seen as a star made by the GOLF (Global Oscillation at Low Frequencies[7]) experiment on SOHO (Solar and Heliospheric Observatory[8]) orbiting at the Earth-Sun L_1 Lagrangian point. Here it can be seen that there is a slight difference in the large separations for even and odd ℓ-modes (cf. $\Delta\nu_0$, $\Delta\nu_1$), that the small separation lifts the degeneracy between modes of (n, ℓ) and $(n - 1, \ell + 2)$, etc., and that there is a small difference between the small separations for even and odd ℓ-modes (cf. $\delta\nu_0$, $\delta\nu_1$).

Ultimately, it is the goal of asteroseismology for any star to detect enough frequencies over ranges in n, ℓ and m that the interior sound speed may be mapped with precision, so that

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[6]http://bison.ph.bham.ac.uk
[7]http://golfwww.medoc-ias.u-psud.fr
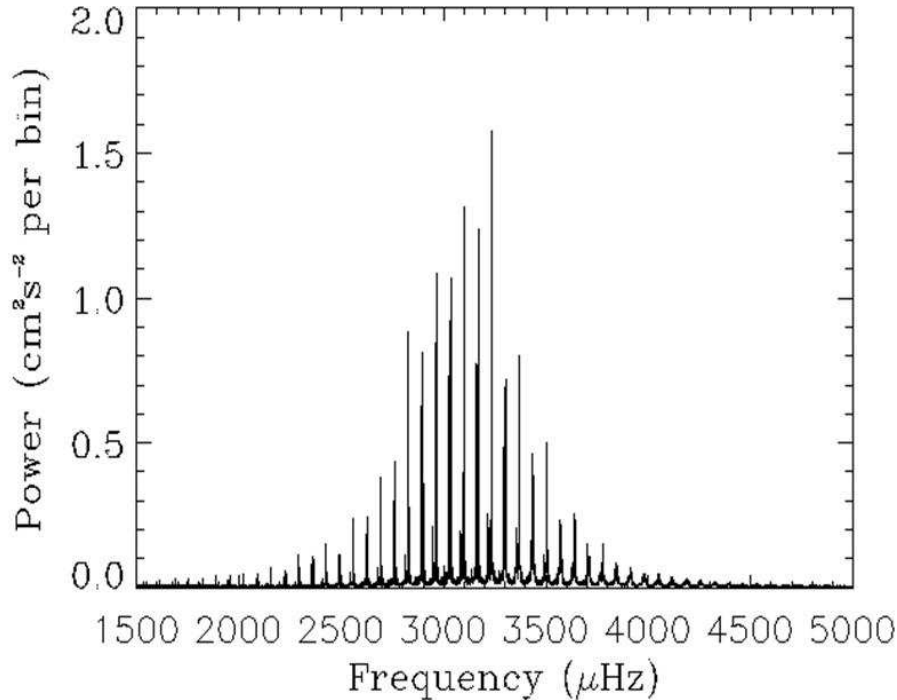[8]http://sohowww.nascom.nasa.gov

Figure 8: This shows a power spectrum of radial velocity variations in the sun seen as a star for 9.5 yr of data taken with the Birmingham Solar Oscillation Network (BiSON) telescopes. The equivalent amplitude noise level in this diagram is $0.5 \, \text{mm} \, \text{s}^{-1}$. Figure courtesy of the BiSON team.

deductions can be made about interior temperature, pressure, density, chemical composition and rotation, i.e. it is the goal to "see", and to see clearly, inside the star. A step along the way is to resolve sufficient frequencies in a star, and to identify the modes associated with them with enough confidence that the large and small separations may be deduced with confidence. That step alone leads to determinations of the fundamental parameters of mass and age for some kinds of stars.

Fig. 10 shows an "asteroseismic HR Diagram" (Christensen-Dalsgaard 1993) where the large separation clearly is a measure of mass (largely because of the relationship between mass and radius), and the small separation is most sensitive to the central mass fraction of hydrogen, hence age. Now that many solar-type oscillators have been found, it is possible to begin to model them using the large and small separations (see Section 7.1). The pattern of high overtone even and odd ℓ modes is also observed in some roAp stars, although their interpretation for those stars is more complex because of the strong effects of their global magnetic fields on the frequency separations (see Section 7.3).

6   A pulsation HR diagram

Fig. 11 shows a black-and-white version of the "pulsation HR Diagram" produced by Jørgen Christensen-Dalsgaard. A much more colourful version of this diagram is frequently presented at stellar pulsation meetings to put particular classes of stars into perspective. As an example, in Section 4.5 it was pointed out that the g-modes are particularly sensitive to the core conditions in the star (see Fig. 7). It is that sensitivity that has made the discovery of g-modes in the
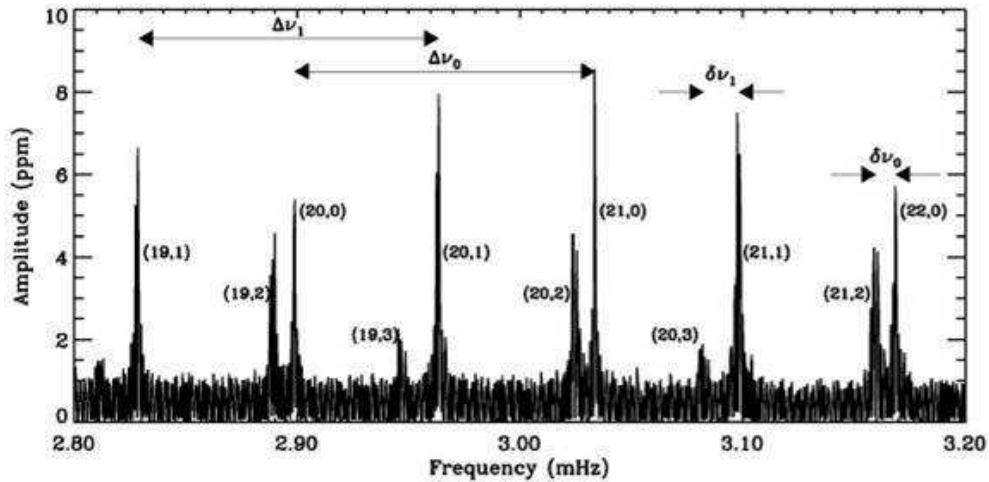
Figure 9: This amplitude spectrum of radial velocity variations observed with the GOLF instrument on SOHO clearly shows the large and small separations in the p-modes of the sun. Courtesy of the GOLF science team.

sun such a long-sought goal – so much so that the discovery of g-modes in the sun has been claimed repeatedly, but general acceptance of those claims is still lacking. On the other hand, g-mode pulsators are common amongst other types of stars – even some, the γ Dor stars, not very much hotter than the sun and overlapping with the solar-like oscillators, keeping hope alive that g-modes may eventually be detected with confidence in the sun. There are three places in Fig. 11 where there are p-mode and g-mode pulsators of similar structure: for the β Cep (p-mode) and Slowly Pulsating B (SPB g-mode) stars on the upper main sequence; for the δ Sct (p-mode) and γ Dor (g-mode) stars of the middle main sequence; and for the EC 14026 sub-dwarf B variables (p-mode) and the PG 1716+426 stars (g mode). Stars pulsating in both p-modes and g-modes promise particularly rich asteroseismic views of their interiors.

6.1   Why do stars pulsate: driving mechanisms

We have looked in some detail now at *how* stars pulsate. But *why* do they pulsate? Firstly, not all stars do. It is an interesting question as to whether all stars would be observed to pulsate at some level, if only we had the precision to detect those pulsations. For now, at the level of the precision of our observations of mmag in photometry and $m\,s^{-1}$ is radial velocity, we can say that most stars do *not* pulsate.

The ones that do are pulsating in their natural modes of oscillation, which have been described in the previous sections. In the longest known case of a pulsating star, that of o Ceti (Mira), we usually attribute the discovery of its variability to Fabricius in 1596. So this star has been pulsating for hundreds of years, at least. In many other cases we have good light curves going back over a century, so we know that stellar pulsation is a relatively stable phenomenon in many stars. That means that energy must be fed into the pulsation via what are known as *driving mechanisms.*

As a star pulsates, it swells and contracts, heats and cools as described in the previous sections. For most of the interior of the star, energy is lost in each pulsation cycle; i.e. most of the volume of the star *damps* the pulsation. The observed pulsation can only continue, therefore, if there is some part of the interior of the star where not only is energy fed into the
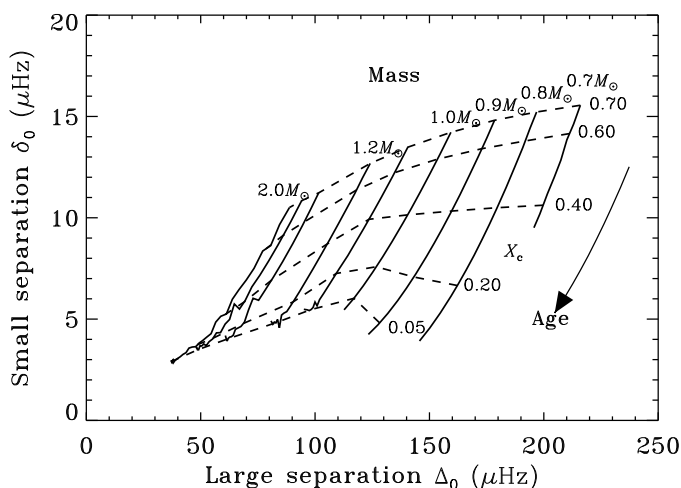
Figure 10: An asteroseismic HR Diagram in which the large separation is most sensitive to mass, and the small separation is most sensitive to age. The solid, nearly vertical lines are lines of constant mass, and the nearly horizontal dashed lines are isopleths of constant hydrogen mass fraction in the core. Figure courtesy of Jørgen Christensen-Dalsgaard.

pulsation, but as much energy is fed in as is damped throughout the rest of the bulk of the star.

A region in the star, usually a radial layer, that gains heat during the compression part of the pulsation cycle drives the pulsation. All other layers that lose heat on compression damp the pulsation. For Cepheid variables, RR Lyrae stars, δ Scuti stars, β Cephei stars – for most of the pulsating variables seen in Fig. 11 – the driving mechanism is opacity, thus it is known as the κ-mechanism. And for the κ-mechanism to work there must be plenty of opacity, so the major drivers of pulsation are, not at all surprisingly, H and He.

Simplistically, in the ionisation layers for H and He opacity blocks radiation, the gas heats and the pressure increases causing the star to swell past its equilibrium point. But the ionisation of the gas reduces the opacity, radiation flows through, the gas cools and can no longer support the weight of the overlying layers, so the star contracts. On contraction the H or He recombines and flux is once more absorbed, hence the condition for a heat engine is present: the layer gains heat on compression.

Of course, since the layers doing the driving are ionisation zones, some of the energy is being deposited in electrostatic potential energy as electrons are stripped from their nuclei, and that changes the adiabatic exponent, Γ. That causes the adiabatic temperature gradient to be small, so these zones are convections zones, too, and variations in Γ can make small contributions to the driving in some cases.

For decades the pulsation driving mechanism for β Cep stars was not understood. Only recently has it been found that atomic diffusion levitates and concentrates enough Fe into an ionisation zone that the κ-mechanism – operating on Fe, not H or He – can drive the pulsation in these stars. Similarly, pulsation in the sdBV stars, labelled as EC 14026 stars (p-mode pulsators) and Betsy stars (g-mode pulsators) in Fig. 11, is driven by the κ-mechanism operating on Fe.

The other major driving mechanism that operates in the sun and solar-like oscillators, as well as some pulsating red giant stars, is *stochastic* driving. In this case there is sufficient
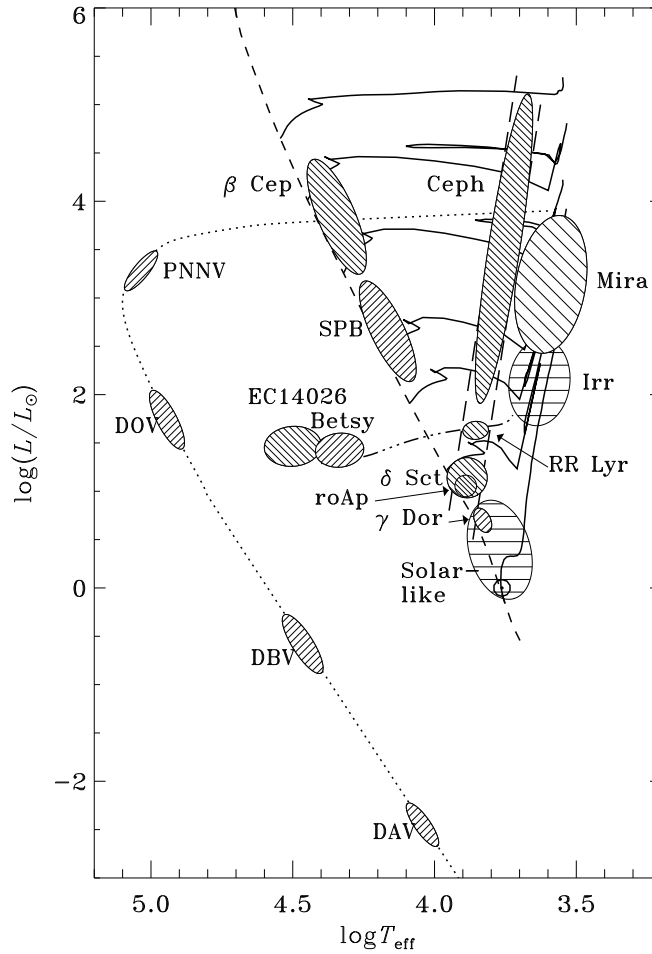
Figure 11: A pulsation HR Diagram showing many classes of pulsating stars for which astero-seismology is possible. Courtesy of Jørgen Christensen-Dalsgaard.

acoustic energy in the convection zone in the star that the star resonates in some of its natural oscillation frequencies where some of the stochastic noise is transferred to energy of global oscillation. In a similar way, in a very noisy environment, musical string instruments can be heard to sound faintly in resonance with the noise that has the right frequency.

The third major theoretical driving mechanism is the ϵ-mechanism, where in this case that is the epsilon from $\mathrm{d}L(r) = 4\pi r^2 \rho(r)\epsilon(r)\mathrm{d}r$. That is, it is the energy generation rate in the core of the star. Potentially, variations in ϵ could drive global pulsations. This has been discussed as a possible driving mechanism in some cases, but there is no known class of pulsating stars at present that are thought to be driven by the ϵ-mechanism.

6.2 What selects the modes of pulsation in stars?

So a star is driven to pulsate by one of the driving mechanisms described above. What decides *which* mode or modes it pulsates in? Why do most Cepheids pulsate in the fundamental radial mode, but some pulsate also in the first overtone radial mode, and rarely a few pulsate only in overtone modes? Why do the sun, solar-like oscillators and roAp stars pulsate in high overtone p-modes? Why do white dwarfs pulsate in high overtone g-modes? What is the mode selection

mechanism in these stars?

These are complex questions for which answers are not always known. Some generalities are: The fundamental mode is most strongly excited for many stars, as it is for musical instruments. The position of the driving zone can determine which modes are excited, just as where a musical instrument is excited will determine which harmonics excited, and with what amplitude. For example, if a guitar is plucked at its twelfth fret (right in the centre of the string), then the first harmonic (which has a node there) will not be excited. You cannot drive a mode by putting energy in a node where that mode does not oscillate. So if the driving zone for a star lies near the node of some modes, those modes are unlikely to be excited.

Any physical property of an oscillator that forces a node will select against some modes, and/or perturb the frequencies and eigenfunctions of the modes. Thus in roAp stars the strong mostly-dipolar magnetic field almost certainly determines that dipole pulsation modes are favoured. In stratified white dwarf stars, the steep gradient of mean molecular weight between layers of H, He and C/O modifies the character of some modes and may select modes. In the stochastically excited pulsators it is the modes that have natural frequencies near to the characteristic time-scale for the convective motions that are excited.

So there is understanding of mode selection, but in many stars the precise answer for why those particular modes are excited is not known, or is incompletely understood. Of course, some physical characteristic of the star is selecting the modes that are excited, or not damped, as the case may be, and a determination of that selection mechanism will allow us a clearer, more detailed look at the interior of the star.

And that, of course, is the goal of asteroseismology.

7 Conclusion: Some selected results from asteroseismology

We have had a good look at what pulsating stars look like, and how asteroseismology probes their interiors. To go beyond the general descriptions of stellar pulsation, to put those descriptions to use so we can see better what they mean, I finish this paper with a presentation of a small, personal (and possibly idiosyncratic) sample of some recent interesting asteroseismic studies. Asteroseismology is rich with successes, and these are but a sampling of a few.

7.1 Solar-like oscillators: α Centauri

For decades searches for solar-like oscillators were made with many claims of discoveries, all of which were later dismissed as over-interpretation of noise. Kjeldsen et al. (1995) claimed to have discovered solar-like oscillations in the G0 subgiant star η Bootis, but with all the false alarms that had preceded this claim, it was not fully believed until it was confirmed by Kjeldsen et al. (2002) some years later. In retrospect, this was the first detection of a solar-like oscillator. In the meantime definitive detection of solar-like oscillations was announced for β Hydri (Bedding et al. 2001) and confirmed by Carrier et al. (2001).

Now there are many solar-like oscillators known, with a rapid pace of discovery of more of them. The best data set currently available is for α Cen A. Bedding et al. (2004) have analysed data obtained by Butler et al. (2004) for α Cen A and found the expected comb of p-mode frequencies reminiscent of that of the sun seen in the BiSON data in Fig. 8. They find a large separation of $\Delta\nu_0 = 106.2\,\mu$Hz and have even succeeded in partially resolving the $\ell = 0$, $\ell = 2$ even modes and the $\ell = 1$, $\ell = 3$ odd modes for estimates of the small separations, $\delta\nu_{02}$ and

$\delta\nu_{13}$, as seen with more resolution for the sun in the GOLF data in Fig. 9. Part of the resolution problem for α Cen A is that the mode lifetimes are only of the order of 2 d. The noise level for the Butler et al. (2004) data is a stunningly low $1.9 \, \text{cm s}^{-1}$! Of course, this is about 40 times larger than the noise in the BiSON 9.5-yr data set in Fig. 8, but then α Cen A is nearly 10^{11} times fainter than the sun and the observations only covered a few days, not 9.5 yr. The precision for the α Cen data is about what the solar astronomers were getting 20 years ago, so this is the state of the art in asteroseismology of solar-like stars. Miglio & Montalbán (2005) give an extensive discussion of their modelling the α Cen system using seismic and other data.

7.2 Planet finding and asteroseismology

The discovery of planets around other stars is an important field in astronomy and is certainly big news. Everyone is interested in these discoveries and the real race is to find Earth-like planets, since our own origins and uniqueness (or not, as the case may be) are of prime concern to us. The main technique for discovering planets and for asteroseismology of many types of stars is high precision spectroscopy to obtain the highest possible precision radial velocity measurements. For this purpose both fields need highly efficient spectrographs and large telescopes. Asteroseismology has the even more stringent need for high time resolution. With the combined demands of high spectral resolution, high time resolution and high signal-to-noise ratio, even for very bright stars observers working with 8-m telescopes are wanting even bigger apertures. Big apertures are needed just as much as for faint object work, but for different reasons.

Over 10 years ago when the first extra-solar planet was discovered orbiting 51 Peg with an orbital period of only 4.2 d, another study suggested that there was no planet at all – that line profile variations could be detected in the spectrum of 51 Peg. The Doppler shift of a star in reflex to an orbiting planet will not change line shape, but stellar pulsation can do so. The implications of the suggested line profile variability for 51 Peg was that it was pulsating in a g-mode with a period of 4.2 d. As 51 Peg is very similar to the sun, and g-modes in the sun are long-sought and long-desired for their ability to probe the solar core, this was an exciting suggestion to asteroseismologists. But it was a horrifying thought to the planet finders, and it generated a lot of excitement. In the end, the line profile variations were not confirmed, and the planet survives (along with a rapidly growing list now of over 150 extra-solar planets[9]). Unfortunately for asteroseismology, there are still no g-modes known in solar-type stars, or in the sun.

The two fields of extra-solar planet finding and asteroseismology work hand-in-hand, both with the technique of ground-based high precision radial velocity studies, and with photometry from satellites – MOST, the Canadian photometric satellite, is in orbit and COROT, the French-led mission is to be launched in 2006. But already there is a fascinating cross-over star: μ Arae (Bazot et al. 2005; Bouchy et al. 2005). This star has at least three planets - two gas giants, and a third which is estimated to have a mass of only 14 Earth masses. *And* it is a solar-like oscillator with over 40 identified modes of degree $\ell = 0$ to 3 with a large separation of $\Delta\nu_0 = 90 \, \mu\text{Hz}$. This is a very exciting object in both fields of research.

7.3 roAp stars

The rapidly oscillating Ap (roAp) stars are H-core-burning SrCrEu peculiar A stars with T_{eff} in the range of about 6600 K to 8500 K that pulsate in high overtone p-modes with periods in the range $5.65 - 21$ min. There are 35 of them known as of late-2005. They are amongst the most
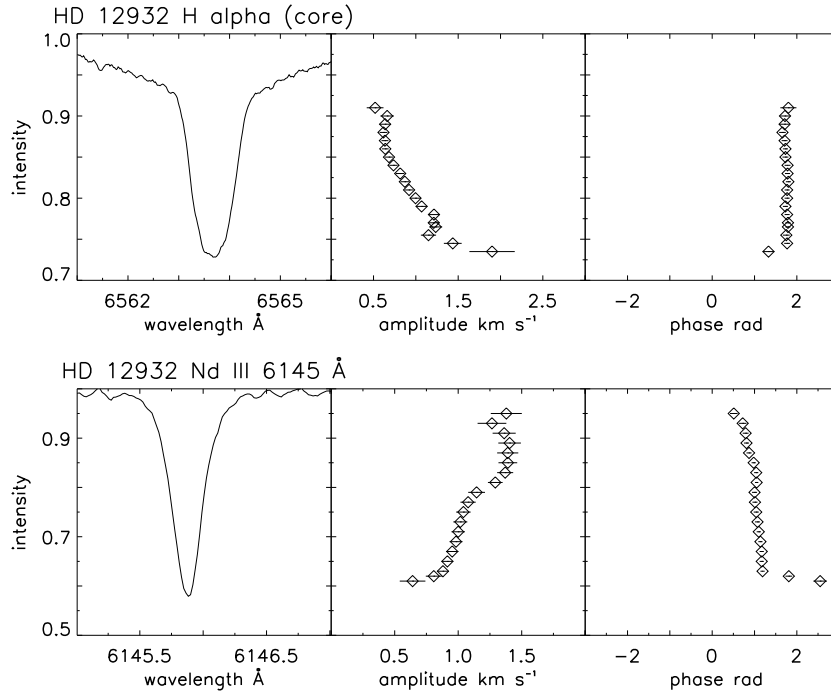
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[9]See: http://www.obspm.fr/encycl/cat1.html

Figure 12: These panels show the line profiles for Hα and Nd III 6145Å for HD12932 in the left panel of each row. The centre and right panels of each row show the amplitudes and phases of the pulsation as a function of depth in the line, obtained from least squares fits of the principal pulsation frequency to the RV variations of the line bisector at each depth. Where the error bars cannot be seen, they are smaller than the data points.

peculiar stars known and even include among their numbers the extreme case of HD 101065, arguably the most peculiar star known.

Radial velocity studies of the pulsations in these stars provide new constraints on their atmospheric structure and element stratification, since the pulsation modes can be resolved in three dimensions as can be done for no other star but the sun. Because of element stratification, Fe is concentrated in the observable layer between $-1 \leq \log \tau_{5000} \leq 0$ and ions of the rare earth elements Pr and Nd are concentrated above $\log \tau_{5000} \leq -5$, while the narrow core of Hα, forms at continuum optical depths of about $-5 \leq \log \tau_{5000} \leq -2$. Thus it is that for the roAp stars we can resolve the pulsation behaviour as a function of optical depth over a large range, $-5 \leq \log \tau_{5000} \leq 0$ and possibly even higher.

Fig. 12 shows how the pulsation amplitude in the roAp star HD 12932 increases with height in the atmosphere through the Hα line-forming layer, then decreases again above that in the layer where the Nd III 6145 Å line forms. Theoretical models of roAp star atmospheres are not yet able to model as high in the atmosphere as the layer of formation of the Nd III lines, hence the turn-over and then decrease in velocity is a surprise. See, for example, the top panel of Figure 8 in Saio (2005) where the amplitude of the pulsation simply increases through this layer.

The roAp stars are *oblique pulsators*: Their pulsation modes are aligned with strong, global magnetic fields that are themselves inclined to the rotation axis of the star, so that over a rotation period it is possible to see the pulsation mode from varying aspect. This is a unique property of these stars that allows a detailed study of the character of their pulsation. One of the best-studied roAp stars is HR 3831. Photometric studies show that its single pulsation

mode is aligned with its magnetic axis and is distorted from a simple dipole $\ell = 1$ mode (see Kurtz et al. 1997). Kochukhov (2005) has analysed several spectral lines in detail over the 2.85-d rotation period of this star and made the first-ever 3-D map of the pulsation for any nonradially pulsating star. Thus, there is rich information in the roAp stars and their future study promises many new discoveries.

7.4   White dwarf pulsators

White dwarf variables are high overtone g-mode pulsators and are the current champions of asteroseismology. They have more frequencies detected than any other type of pulsating star, other than the sun, and theory has been more successful in extracting astrophysical information for them than for any other kind of pulsator. As can be seen in Fig. 11 there are three main regions of white dwarf pulsation: The DOV, DBV and DAV stars, where the nomenclature is D = white dwarf; V = pulsating variable; and O, B and A refer to spectra that resemble O, B and A stars in the presence of He and H lines.

The best studies of pulsating white dwarfs have been carried out by the Whole Earth Telescope, WET. The WET website contains a wealth of information and references to published papers from many extended coverage (Xcov) campaigns[10]. An outstanding example is their study of the DOV star PG 1159-035 (Winget et al. 1991) where they found 101 independent pulsations modes. Models yielded a mass of $M = 0.586 \pm 0.003\,\mathrm{M_\odot}$; independent tests using distances determined from parallaxes and the mass-radius relation indicate that the quoted precision is probably correct. The periods in PG 1159 are in the range $385 \leq P \leq 1000\,\mathrm{s}$; they are high-overtone ($n >> \ell$) g-mode pulsations, as is the case for other pulsating white dwarfs. Asymptotic theory gives a clear prediction of period spacing for such stars (cf. eq. 10 in Section 4.5), and deviations from that are used to derive the compositional stratification in their atmospheres, i.e. the mass of the surface He and/or H layers - possibly even resolving He^3 and He^4 layers (Wolff et al. 2002). PG 1159 clearly shows $\ell = 1$ and 2 modes, but not $\ell = 3$. The magnetic field strength is less than $6000\,\mathrm{G}$ – a very small value for a white dwarf star where field strengths are often MG. Clearly asteroseismology of white dwarf stars is highly successful in extracting astrophysically interesting information.

One of the more striking properties of white dwarfs being studied by asteroseismology now is the C/O interior composition and the potential crystallisation of the core in the most massive of the DAV stars, such as BPM 37093 (intensively studied by WET; see Kanaan et al. 2005), into Earth-sized "diamonds" (see Metcalfe et al. 2004). These new kinds of "diamonds" are C and O in a partial to complete crystalline state of degenerate matter; this state of matter has been (briefly) produced experimentally using petaWatt lasers at Lawrence Livermore Laboratory. So next time you think of "Twinkle, twinkle, little star ...", think about the bizarre reality of diamonds in the sky.

Perhaps even more interestingly (if there can be something more interesting than an Earth-sized diamond!), white dwarfs produce much of their total radiation in neutrinos; for the very hot white dwarfs the neutrino flux exceeds the photon flux. Some of the neutrinos are brehmsstrahlung neutrino pairs from electrons, but most are plasmon neutrinos that are created by photons that have an effective rest mass when travelling through the dense plasma in the white dwarf and can decay into neutrino-antineutrino pairs. Thus the cooling rate for white dwarfs is a measure of the neutrino generation rate and asteroseismology can test the cooling rate. This field is young; see O'Brien & Kawaler (2000) for a first attempt at a test of standard lepton theory in a dense plasma from asteroseismology of the DOV star PG 0122+200.

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[10]http://wet.physics.iastate.edu/

7.5 sdBV stars

The sdBV stars are both p-mode pulsators (the EC 14026 stars) and g-mode pulsators (the PG 1716+426, or Betsy stars). They are extreme horizontal branch stars, essentially He stars with very thin H surface layers. Their pulsation periods are typically $100 - 200\,\mathrm{s}$, although periods up to $500\,\mathrm{s}$ are known, for the EC 14026 stars; and $45\,\mathrm{min}$ to $2\,\mathrm{hr}$ for the Betsy stars. Their photometric amplitudes are typically only a few percent, although in the case of PG 1605 it is as high as $0.2\,\mathrm{mag}$. They were discovered observationally (Kilkenny et al. 1997) and predicted theoretically (Charpinet et al. 1996) independently, and at the time.

One of the most exciting of the sdBV stars is PG 1336−018. This star is a multi-periodic, high overtone g-mode pulsator with periods in the $170 - 200\,\mathrm{s}$ range, and is an eclipsing binary with an orbital period of $2.42\,\mathrm{hr}$. The combination of the short pulsation period and the eclipses makes the light curve of this star particularly striking. See Figure 1 of the WET campaign on this star in Kilkenny et al. (2003).

In the second chapter of the well-known textbook, *Nonradial Oscillations of Stars* (Unno et al. 1996) the authors discuss the advantages of finding nonradial pulsators in eclipsing binary systems, since the change in amplitude and possibly phase during the eclipse potentially allows the mode to be identified. Also, pulsating stars in close binary systems offer the possibility to study tidally induced oscillations. Both of these make PG 1336−018 a particularly interesting object.

To see what detailed information is being extracted asteroseismically from sdBV stars, Charpinet et al. (2005) have studied PG 1219+534 and derived many important parameters, including its mass ($M = 0.457 \pm 0.012\,\mathrm{M_\odot}$) and the mass of the residual H atmosphere ($\log M_{\mathrm{atm}}/M_{\mathrm{total}} = -4.25 \pm 0.15$), showing that this really is a He star with very little H left. The lack of rotationally split m-modes even suggests that the star is rotating very slowly. The modes are identified as consecutive overtones of degree $\ell = 0, 1, 2, 3$. Thus, exceptional knowledge of this important stage of stellar evolution is illuminated by asteroseismology.

7.6 β Cephei stars

Fig. 11 shows that the β Cep stars are B0 – B2 main sequence stars; they pulsate mostly in p-modes, but also in g-modes. Remarkable studies of two β Cep stars have recently been made. Aerts et al. (2003) found for HD 129929 six frequencies: three were identifiable as the m-modes of a dipole triplet, and from the frequency spacings two were identified as two m-modes of a quadrupole quintuplet, and one was a radial mode. Models of these modes show that from the different spacing of the rotationally split modes, the core must be rotating faster than the surface. This is the first detection of differential rotation with depth for any star other than the sun. The authors were also able to make an estimate of the amount of overshooting in the convective core.

The β Cep star ν Eri has been studied for decades (Aerts et al. 2004; Jerzykiewicz et al. 2004; De Ridder et al. 2004), showing the presence of both p-modes and g-modes, with rotationally split m-mode triplets. From these observations Pamyatnikh et al. (2004) found an increased core rotation rate compared to the surface rate, as in HD 129929. Ausseloos et al. (2004) have modelled these same observations and found that no standard B star model can explain the pattern of observed frequencies in this star. An increase in the Fe abundance, perhaps throughout the star is needed. Thus the β Cep stars are yielding some of their interior secrets better than any other type of pulsating star.

8 A last word

Eddington's "appliance" to look beyond the barrier of stellar surfaces and *see* the interiors of stars is asteroseismology. The secrets it has revealed already are stunning, but the field is young, and, no doubt, the best is yet to come.

Acknowledgements

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[11]http://www.eneas.info/

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