Polytropes as Simple Models of β Cephei Stars

M. Godart [1], R. Scuflaire[1], A. Thoul[1] and A. Noels[1]

[1]Institut d'Astrophysique, Université de Liège, Belgique

Abstract: β Cephei stars have a simple structure: a convective core surrounded by a radiative envelope. It is therefore worth trying to describe β Cephei stars with composite polytropes which are useful to retrieve structure parameters from frequency spectra. We show that the structure of β Cephei models can relatively well be described with two-zone polytropic models. However this description is not convincing to depict oscillations of β Cephei models.

1 Introduction

β Cephei variables have recently been the subject of several papers and reviews (Dziembowski & Pamyatnykh 1993; Pamyatnykh 1999). These stars are population I stars with spectral type B0 to B4. They are 8 to 12 $M_\odot$ stars in their main sequence phase of evolution. These stars mostly pulsate in the fundamental radial mode and some also show small degree l non radial pulsations. They have periods from 2 to 8 hours with amplitude from 0.01 to 0.3 magnitude. The driving mechanism of the pulsation is the κ mechanism due to the iron opacity bump.

2 Structure of β Cephei Models

β Cephei stars have a rather simple structure: they are made of a convective core (c) surrounded by a radiative envelope (e). The convection in the core is adiabatic, so that the inner part of the star has a polytropic structure of index 3/2, i.e., there is a simple relation between P and ρ which reads $P = K\rho^{1+1/n}$ with a polytropic index $n = 3/2$ and K a constant (Chandrasekhar 1957; Schwarzschild 1958). If the structure of the envelope is close to a polytropic structure, β Cephei models could then be described by two-zone polytropic models. These are very useful as simple examples, while not being too far from realistic models. It is generally said that massive main sequence models (Fig. 1, thick line) are not too far from polytropic models of index 3 (the standard model), so we try to represent envelopes by polytropes of index 3. We calculate four evolution sequences of β Cephei stars, of 8, 9, 10 and 11 $M_\odot$; each sequence has a chemical composition $X = 0.7$ and $Z = 0.02$ and no overshooting. The lines corresponding to the two different polytropic indexes are plotted in a $\log P$ vs. $\log \rho$ diagram (Fig. 2). Dashed and dotted lines stand for the polytropic index of the core, 3/2, and the envelope, 3, respectively; three models of the 11 $M_\odot$ sequence are also shown (solid lines). These models are picked out from the beginning (model 1) to the end of the main sequence stage (model 3), before the second gravitational contraction, while the second model is in the middle of the main sequence phase (Fig. 1). Some of their properties are given in table 1. At the beginning of the main

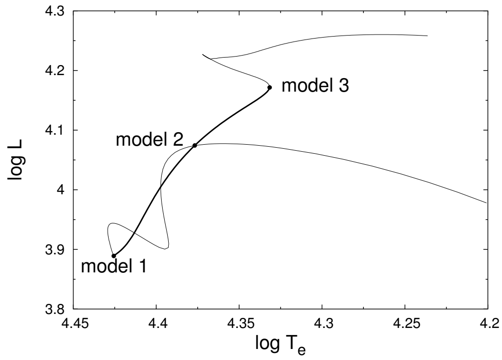

Figure 1: HR diagram of the 11 $M_\odot$ sequence.

	X_c	$\log \frac{\rho_c}{\bar{\rho}}$	$\frac{\mu_c}{\mu e}$	q_c
Model 1	0.6976	1.582	1.002	0.3164
Model 2	0.3083	2.132	1.434	0.2169
Model 3	0.0425	2.724	2.032	0.1500

Table 1: Some properties of the 11 $M_\odot$ models: hydrogen abundance X_c, mass concentration $\log \rho_c/\bar{\rho}$, mean molecular weight ratio between the core and the envelope μ_c/μ_e and convective core mass fraction q_c.

sequence of β Cephei models, the mean molecular weight in the core is equal to the one in the envelope, and the corresponding polytropic model has $\mu_c/\mu_e = 1$. The mean molecular weight ratio increases as the models evolve. In Fig. 2 we see that convective cores have indeed a polytropic structure and the radiative envelopes seem to have a polytropic structure of index 3. The convective core appears at the top right part of the figure. The core and the envelope in β Cephei models are separated by a small region of mean molecular weight gradient created by the receding core during the main sequence phase (Fig. 2, thick line). Analogous results are obtained for the other sequences of 8, 9 and 10 $M_\odot$.

We have shown that envelopes of β Cephei models can be represented by polytropic models. It is important to point out that the polytropic models are usually characterized by four parameters: the mass, the radius, the mean molecular weight ratio and the convective core mass fraction. However the mass and the radius of one model are obtained by a homologous transformation from another model, so that the polytropic models depend in fact on two parameters only: μ_c/μ_e and q_c.

We generated a grid of 36 polytropic models by varying these two parameters. In Fig. 3 we represented each polytropic model with lines of constant mass concentration (solid lines). We have also plotted the four evolution sequences, as well as their lines of constant mass concentration ($\log \rho_c/\bar{\rho} = 1.8$ to 2.8; solid lines with full circles). We see in this figure that the density of β Cephei models do not behave exactly like the density of polytropic models.

3 Oscillations of β Cephei Models

Polytropic models can approximately model the structure of a β Cephei star, but do they also describe its oscillations? Frequencies of polytropic models depend on three parameters: q_c, μ_c/μ_e and GM/R^3. We get rid of the third parameter by taking frequency ratios. We want to find a relation between frequency ratios and the parameters of polytropic models.

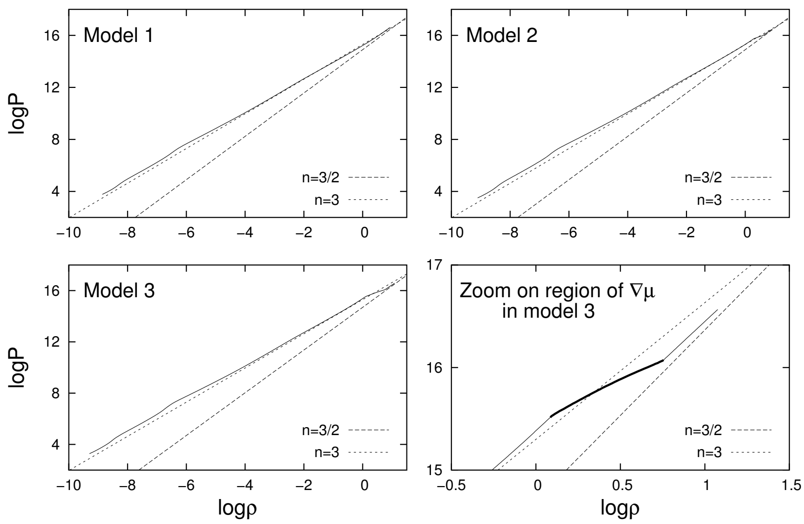

Figure 2: $\log P - \log \rho$ diagram of 3 models of the 11 $M_\odot$ sequence (solid lines). Dotted lines and dashed lines represent polytropic models of index 3 and 3/2, respectively (see text).

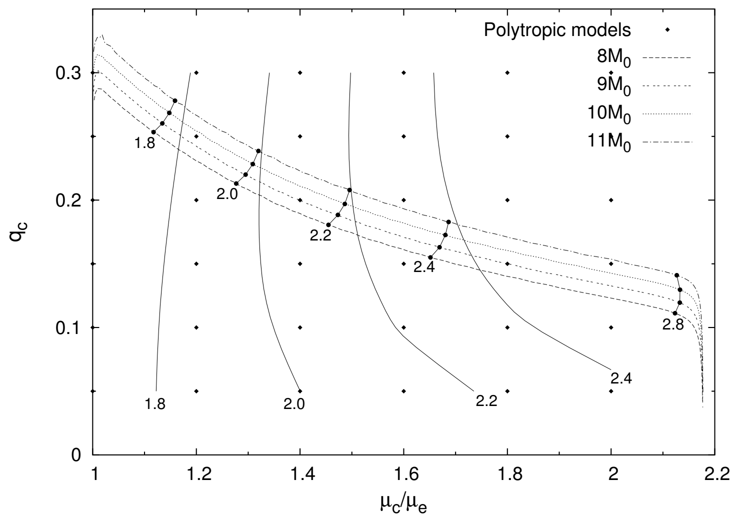

Figure 3: $q_c - \mu_c/\mu_e$ diagram. A grid of 36 polytropic models is shown (diamonds) with curves of constant mass concentration (solid lines) labelled by $\log \rho_c/\bar{\rho}$. The main sequence of 8 to 11 $M_\odot$ is shown (no-solid lines) with their curves of constant mass concentration (solid lines with full circles), labelled by $\log \rho_c/\bar{\rho}$.

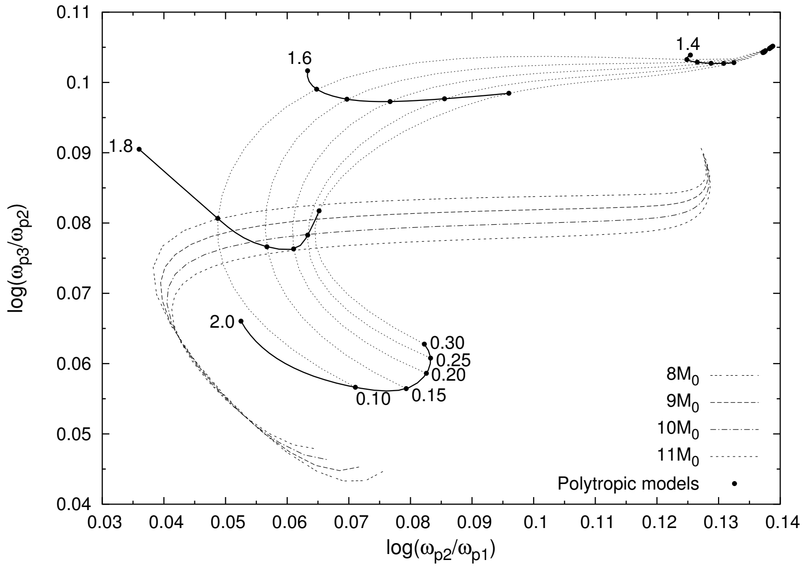

Figure 4: Frequency ratios of polytropic models (circles). Lines of constant mean molecular weight ratio μ_c/μ_e (solid lines) labelled by μ_c/μ_e and lines of constant convective core mass fraction q_c (dashed lines) labelled by q_c from polytropic models are displayed. Different lines represent the frequency ratios of the $l = 1$ modes of the main sequence of 8 to 11 $M_\odot$.

Fig. 4 represents one of the most exploitable relations between frequency ratios: a $\log \omega_{p3}/\omega_{p2}$ vs. $\log \omega_{p2}/\omega_{p1}$ diagram, computed for $l = 1$. These frequency ratios define a good grid; placing in this grid an unknown model can provide the values of its parameters μ_c/μ_e and q_c. In Fig. 4 we show the polytropic models as circles and the lines of constant μ_c/μ_e and q_c. The frequency ratios of the sequences of 8 to 11 $M_\odot$ are also shown, the first models at the top and the right of the figure describe the beginning of the main sequence. Along the evolution of each sequence, the convective core mass fraction seems to be almost constant while the mean molecular weight ratio increases.

Fig. 4 shows qualitative similarities between the behaviour of physical and polytropic models. However it is clear that the curves determined for polytropic models are of no use for physical models, they cannot help retrieving the q_c and μ_c/μ_e parameters of a physical model from the frequency ratios. Refinements of the polytropic models such as adopting other values of the polytropic index in the envelope, truncating the model at a non-vanishing pressure to better match photospheric conditions or varying the Γ_1 coefficient do not lead to a better agreement of the $\log \omega_{p3}/\omega_{p2}$ vs. $\log \omega_{p2}/\omega_{p1}$ diagram. However this sort of diagram, calibrated with physical models, could be used to obtain q_c and μ_c/μ_e from observed frequencies of real β Cephei variables.

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